Mathematical anxiety and the Zone of Proximal Development

Kyriakos Petakos

Abstract. In the recent article mathematical anxiety in a group of junior students is discussed. These students have just entered tertiary education in the field of Hotel Management and have to deal with Business mathematics during their first semester. A question regarding mathematical anxiety is posed to them by the instructor. Their answers are written down and a sociocultural analysis ensues based on the renowned Vygotsky's theory.

Key words. Mathematical anxiety, Zone of Proximal Development, Sociocultural theory, Teaching mathematics.

Sommario. Nell'articolo viene discussa l'ansia matematica in un gruppo di studenti junior. Questi studenti sono appena entrati nell'istruzione terziaria nel campo della gestione alberghiera e hanno a che fare con la matematica aziendale durante il loro primo semestre. Una domanda riguardante l'ansia matematica è posta loro dal docente. Le loro risposte vengono trascritte e ne consegue un'analisi socioculturale basata sulla famosa teoria di Vygotskij.

Parole chiave. Ansia matematica, Zona dello sviluppo prossimale, Teoria socioculturale, Insegnamento della matematica.

Introduction

Mathematical anxiety does not require any special definition as it is encountered in our every day teaching practice. It is a sort of anxiety that impedes the process of learning and leaves to us the difficult task of embellishing and beautifying mathematics as a subject material. Especially when teaching to university students that have already created their convictions about the subject in contrast with students at younger ages that appear easier to change stance towards mathematics (Petakos, 2016).

In this direction a cornerstone of the sociocultural theory inaugurated by Vygotsky and exemplified bu others (Kozullin, 2015, Chaiklin, 2003), the so-called Zone of Proximal Development, henceforth ZPD, will act as a catalyst. What is the Zone of Proximal Development? A classical definition will ensure " the distance between the actual development level as determined by independent problem solving and the level of potential development developed as determined through problem-solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86 as cited in Chaiklin, 2003, p.40).

In the course of history this notion of ZPD has attracted the interest of many researchers and has adopted interpretations in general similar to each other, albeit with a separate nuance that seems to depict the author's personal stance towards it. For example, Palinscar (1998) advocates that it is "probably one of the most used and least understood constructs to appear in contemporary educational literature" (as cited in Chaiklin, 2003, p. 41).

Our goal here will be to construe how a certain group of students react to the mathematical anxiety employing as much as possible the concept of ZPD. The concomitant experiment will illustrate what we aim at based on this magnificent powerful tool that Vygotsky exhorts to use, language.

Methodology

In a class of 25 students that have just entered the university through a strict examination system, the following question is posed

Have you felt or are you still feeling mathematical anxiety? Deliberate to what or to whom this can be attributed.

Someone might claim that the ensuing writing down of answers will be proven to be biased by the nature of the question. On the contrary, we have to avow here that we exhibit as much loyalty as possible to the sociocultural theory of Vygotsky, one of whose primary components is speech, language.

Our question above is posed in an interpretative manner (Di Martino & Zan, 2010), which empowers the potential researcher and the classroom teacher to realize motives of deliberate actions successfully. We can state that, what we have accomplished here by writing down the students' answers and analysing them, tends to the three dimensions presented in Di Martino and Zan (2010), i.e.

- Emotional disposition toward mathematics
- Vision of mathematics
- Perceived competence of mathematics

That is why we chose to work with a cornerstone of Vygotsky's theory, the ZPD. This approach fits in our role as simultaneously teachers and researchers and moreover provides the student with the ability to give vent to some of their deep-rooted dormant beliefs and attitudes. Demeanours, over which they have mulled throughout the years, but never hitherto have been provided with the opportunity to openly articulate. Albeit the fact that the three dimensions mentioned above are inextricably interconnected, we need to avow that the first two eclipse the third in our impending analysis, without of course neither eliminating it nor diminishing its importance.

The eventual reader might ponder a little on the aforementioned comments and dither about which way to construe the gist of speech in writing down answers. It is a speech with ourselves what we deliberate in our minds before it can be articulated in words. A process in which every fibre of our spiritual being is employed and definitely involved. So, are we reversing the principle of Vygotsky's (Vygotsky, 1978) theory that the interpsychological level precedes the intrapsychological?

No way can that happen. As we grow up, we adapt more and more easily to a set of rules set by the society itself, cultural or not. Deliberation before talking is a rule given to us at an early age, whose repercussions will be experienced in our later life. Ergo, by letting the students write what

they think about the question, we exhort them to this sort of internal speech that will not go unheeded as long as we analyse its aftermath.

Main Article

Out of the 25 students that were asked the aforementioned question, we had a return of 7 after a week of deliberation. Someone might consider poor the participation of the students but remaining loyal to the development of higher psychological processes under the Vygotskyan clout, (Vygotsky, 1978). This can be ascribed to the degree of maturity these students have mastered through the years, which is not always compatible with the one anticipated according to their learning outcomes. All but one seem to acknowledge the existence of mathematical anxiety. A categorization will ensue reflecting on the factor that causes them this anxiety which will also encompass the negative answer.

According to their answers the prevailing factor for the mathematical anxiety appears to be the teacher. Then the student herself and the prejudice surrounding this teaching subject. We will not deal with the educational system-process itself because it is interwoven in their answer with the role of teacher. Let us see how the answers concerning the teacher shape students' demeanour pertaining to this vigorous participant of the educational process.

Answers pertaining to the teacher

- Even if we consider that this anxiety has been always there, then this one manifested itself during middle school, in the face of our teacher of the time. In brief, the mathematical fear had become a person and was moving among us.

- In the end, an equally important role play the professors, who many times are indifferent to their students' understanding, try to cover long teaching material in a short period of time, and as a result, big and uncovered gaps are created among the students.

- Moreover the teachers that are responsible for the lesson do not provide the necessary basis because they have little time at their disposal that is why they focus on the good students.

- Sometimes it is a mistake on behalf of the teachers, who do not dedicate the required time to the students.

- (*This last one is coming from the student who feels liberated from any kind of mathematical anxiety and therefore should be construed differently*). A teacher should be as good a scientist as an educator with the whole meaning of the word, because ultimately it is a nice subject, which with the right teacher would be popular to everybody.

Let us expatiate on the above-mentioned answers. We take into account the first one. Here the student personifies the primary cause of mathematical anxiety, the fear itself, as the mathematics teacher. The space component of the Zone of Proximal Development (symbolically ZPD) manifests itself here. The use of the verb *move* also bestows a dynamic dimension to this space component.

The way that the student expresses herself demonstrates a concept by which we could conceptualise a reverse process of the ZPD. In this example, the presence of a more

knowledgeable experienced peer proves to be to the detriment to the student's understanding of the teaching material. Inspired by this and adhering to our favourite mathematics terminology, we can liken the ZPD to an open interval, whose endpoints can be shifted according to appropriate use. It is sort of a topological interpretation.

The remaining answers touch a very sensitive theme of modern day education. Time allocated to teachers to cover their material according to the existing curriculum. Here the students' demeanour can be construed as beneficial not only to the teacher but also to the educational process itself. It is what we denominate as the time dimension of the ZPD. Allocation of time surely contributes to shaping our zone of proximal development whose direct repercussions are experienced by both the teaching subject and the appointed peer next to her.

Answers pertaining to the student herself

- Naturally a share of responsibility falls upon us

- There are students who are indifferent to many classes and in general to the school. Later on, as it is proven, these students face problems.

- It is just an aspect of my nature.

The second group of answers sheds light upon a self-reproach attitude towards mathematical understanding and its concomitant anxiety. To us, it is of paramount importance that there are students aware of their own share of responsibility, as stated so clearly in the first answer, and do not make an effort to exonerate themselves. Lack of time really deprives us of the chance to interview one by one these students, but we dare construe such a demeanour as a result of their own educational history. It is internalized in the center of the open interval of the ZPD the conviction that failure cannot be ascribed only to external factors, but also to internal ones.

Answers pertaining to the prejudice surrounding the teaching subject

- Of course, it is not mathematics what really causes me anxiety, as much as the way they have passed it on me subconsciously.

- Among the most important factors that predispose me negatively towards mathematics, is the general prejudice that surrounds them and their "enormous" difficulty. To this, parents contribute to an extent, who, considering this learning subject unattainable, force their offspring from an early age to seek help from a teacher outside the school, which consequently increases their anxiety.

In this case we see how contemporary the sociocultural theory of Vygotsky (Vygotsky, 1978) appears to be. The first student enunciated that with a very descriptive way. *The way they have passed it on me subconsciously.* Pass on really highlights the idea to what extent social and cultural processes have affected the student's personal stance. The learning subject cannot remain unaffected and untouched by what transpires around her. The ZPD makes its appearance again, because we have a transmission of messages by either a teacher, or a parent, or a schoolmate at an older age etc. Whether the ZPD enhances or dwindles the student's abilities depends on the activity this more experienced peer carries out within that zone.

The second example demonstrates better the aforementioned role, where the more experienced peer bears the name of parent. The family environment fits in the space component of the ZPD we discussed at the beginning of our main article. Parental intervention and its concomitant repercussions, which could form the inspiring idea of developing another article. Here we can also detect the transitivity property of the ZPD (Petakos, 2016) viewed as an equivalence class

of influences. A parent ill-disposed towards mathematics, viewing it as something unattainable, passes on her child this preoccupation and exhorts her to seek help from a teacher outside classroom. Ergo, the child resorts to this teacher, who in her turn will be affected by the situation, in the sense that her beliefs about how to teach mathematics may be consequently altered. On this equivalence class characterisation, we will provide an explanatory comment in the Conclusions section that will ensue.

Ending this paragraph we need to state a comment given by the student who provided us the very first answer at the beginning of our section.

- − Adoring the ancient Greek language, I understood that mathematics is directly associated with it and especially with its syntactical analysis. Now, as a student I am not afraid of mathematics.

The ZPD can encompass not only humans, but also theoretical abstract objects, which serve as sort of an exhortation source for further learning and development. The ancient Greek language, with its syntactical rules, is a strong incentive for the student to overcome the impediment already posed to her, the mathematical fear occasioned by a middle school teacher. Proper language teaching, another teaching discipline that might not seem akin to mathematics, induces the student to dedicate further time to master this initially fear-causing subject. She feels finally, as a college student, liberated from mathematical anxiety.

Conclusions

Before starting this section, we would like to declare that we were not able to transfer every inspiring phrase that students included in their answers. This can deprive the eventual reader by the generating feeling that these answers might pass on, through which sociocultural theory and its Zone of Proximal Development manifest themselves. We try to make up for it by employing this unstated part of the answers and also the dialogue that ensued the experiment in the Conclusion. To assuage the feelings of those who advocate the prevalence of certain disciplines over others, we avow that mathematics does not monopolize the concept of ZPD (Chaiklin, 2003) but it undoubtedly encompasses it.

In mathematical terminology, the zone of proximal development resembles that of an open interval in its topological meaning. The students is located in that interval ready not just to assimilate knowledge but also to strengthen and reinforce her mental capabilities. Social interaction will suitably serve this purpose by moving the endpoints of the interval. The wider the interval is, the more effective the way her mental capacities will be affected and therefore expanded. The social gift (Silvonen, 2010, p.50) is the shift of the interval's endpoints. Regarding the giver, we can apply this term rather to the teacher, the more competent person of the ZPD, whereas the process of expanding the interval's endpoints can be attributed to a joint effort by the giver and the receiver.

We think by Chaiklin's generality assumption of the ZPD (Chaiklin, 2003) that mathematics is by nature a subject that favours development, the transition from a lower to a higher layer, from the rudimentary form of ε-δ definition of the limit in a Calculus course to the concept of neighbourhood in the topology. Ergo, students' fear grounded on the premises of prejudice as reflected on their answers and the ensuing dialogue really encompass this characteristic of Vygotskyan thought. Riding a bicycle is not the quiddity of the generality principle of the

Vygotskyan pedagogical mind. But the use in general of wheel-based vehicles might approximate the so-called generality. Even if that example may be proven to be somehow unsuccessful, mathematical learning divided into layers corroborates better the aforementioned aspect. Missing a layer should surely form an impediment for moving to the next one, the higher one, and this gap will inexorably mark the student's ability to master mathematics. Building a step by step approach to get a better understanding of the mathematics' gist can surely be of some genuine benefit to the student, the teacher, the microsociety of the school itself. Let alone it complies with ZPD's generality aspect.

The role of a good teacher in the class and its concomitant repercussions on the individual student's learning is associated with the assistance assumption of ZPD (Chaiklin, 2003). We are surely embedded in a classroom that presupposes the existence of a teacher. In the general gist of the word teacher, someone who is not necessarily certified but is surely more competent than the student subject of learning. This can be well represented by a teaching assistant, a talented student, someone who the students can be aware of her competence and are ready to acknowledge that in class. We will remain a little bit on the word competent, since language is of paramount importance to Vygotskyan theory. Students that are advocates of a good teacher may not have the slightest idea of this teacher's knowledge credentials. In fact, in only one answer we viewed the teacher's scientific knowledge along with her performance in the classroom to be of equal importance. In the rest of the answers emphasising on the role of the teacher, students zero in on the interaction between the teacher and themselves. The teacher's educational background, albeit a prerequisite for becoming a teacher, is eclipsed by the calibre of the face-to-face collaboration with the student.

Ending this paragraph, let us reiterate a comment that was introduced in a previous article of ours referring to the ZPD as an equivalence class. It is reflexive by definition. It is inexorably symmetric, given the centre of this zone if the Vygotskyan definition implies such a centre. It is not only to the benefit of the less experienced one but also to the advantage of the more knowledgeable one, which encompasses the principles of modern didactics that in the learning process all involving sides are active. The students learn by interacting with us, and we do enhance our teaching performance by coping through a dialogue with their understanding impediments in the field of mathematics. This reciprocal relationship defines the symmetrical property of the zone of proximal development. Finally, the interaction of two different zones, as presented in the answer that included the parental intervention, demonstrates the transitivity, which completes our definition of an equivalent class.

Declaration of Conflicting Interests

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Acknowledgements

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About the Authors

Kyriakos Petakos
Director of the Advanced School of Tourism Education ASTER
Dimokratias 2, 85100 Rhodes Greece
kyriakospetakos66@gmail.com

Dr Petakos is a mathematician graduated with Excellent from the University of Athens in 1988. He attended graduate courses at the University of Augsburg Germany and completed his PhD at the University of Athens in Applied Probability with Professor Papastavridis. His interests turned to mathematics education, and in this direction, he visited financially covered the SUNY Fredonia and SUNY Geneseo delivering speeches on his research on math education. He is currently Assistant Professor and Director of the Advanced School of Tourism Education of Rhodes ASTER.