article

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Basic geometric-trigonometric equalities and problem-solving: Angular Elements

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Abstract. *This paper presents some equalities that we consider useful for the resolution of general high school level geometry problems. The presented equalities involve only angular elements. The objective of this work is to monitor, on a small scale, the skills and knowledge acquired by students in the eleventh year of school in an Italian region named Apulia. This goal has been done by mean of different exercises where 21 students were asked to recognise and combine the equalities presented. Results are reported and discussed.*

Key words. *Mathematics, Teaching, Geometric equalities, Trigonometric equalities, Angular elements, Problem solving.*

Sommario. *In questo lavoro si presentano alcune uguaglianze che riteniamo utili per la risoluzione di problemi di carattere geometrico a livello di scuola secondaria superiore di secondo grado. Le uguaglianze presentate coinvolgono solo elementi angolari. L'obiettivo del lavoro è quello di monitorare, in scala molto ridotta, il grado di competenze acquisite dagli studenti dell'undicesimo anno di scolarità nella regione italiana denominata Puglia. Questo obbiettivo è stato raggiunto tramite diversi esercizi nei quali è stato chiesto a 21 studenti di riconoscere e combinare le uguaglianze proposte. In conclusione, i risultati sono stati presentati e discussi.*

Parole chiave. *Matematica, Didattica, Uguaglianze trigonometriche, Uguaglianze geometriche, Elementi angolari, Problem-solving.*

Introduction

This article is part of the research previously presented in the first phase (Ligouras, 2017). The first phase of the research considered the learning of a group of students who attended a course on geometric equalities concerning only linear elements. The current paper initially presents a collection of equalities that used correctly favour the resolution of many problems of geometry and trigonometry. In this second phase, geometrical and trigonometric equalities were investigated that involved only angular elements.

Generally, the learning of concepts involving angular elements represents a higher difficulty for students than learning concepts that involve only linear elements.

The equalities have been experimented with groups of students of the third (L11) and of the fourth year (L12) of some high schools of the Puglia region, during their preparation for the mathematical competitions, in the last five years.

The primary purpose of the research was to understand if the Italian school:

- prepares students for the acquisition of skills related to problem-solving;
- offers an adequate degree of acquisition of skills related to problem-solving;
- offers the opportunity for students to build meaningful learning in mathematics.

Furthermore, the students' aptitude to work in small groups of three monitored in tasks requiring knowledge not studied at school.

The difference in performance between male and female pupils also observed.

The students involved in the experimentation described below are different from those involved in the first article already presented.

All the students who took part in the experimentation have a passion for mathematics, high performances and the right motivation to improve them.

Description

Below is a summary of the activities performed and evaluated for this experimentation.

Notations

The figure (Fig.1) shows the annotations useful for reading the basic geometric equations that follow. The annotations provided to each student who participated in the experience.

notations	
A, B, C	vertices of a triangle
a, b, c	sides *BC*, *CA*, *AB*
α, β, γ	angles of a triangle

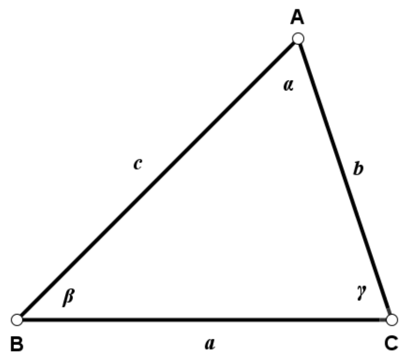

Fig. 1 – Notations

Angular elements

To perform the experimentation, we used the following 40 geometric equations as a material. This collection has been created over the past five years by examining the formulas used to create the problems of international mathematical competitions. For their collection it was very useful Internet but also some documents, articles and books (Andreescu & Feng, 2004, Anonymous, 1904, Altshiller-Court, 2007; Bottema et All., 1969; Engel, 1998; Hang & Wang, 2017; Hobson, 2005; Larson, 1983; Ligouras, 2008; Prasolov & Tikhomirov, 2001; Shariguin, 1989; Zeitz, 2006).

basic angular elements	
b.1	$\sin\alpha+\sin\beta+\sin\gamma=4\cos\frac{\alpha}{2}\cdot\cos\frac{\beta}{2}\cdot\cos\frac{\gamma}{2}$
b.2	$\sin\alpha+\sin\beta-\sin\gamma=4\sin\frac{\alpha}{2}\cdot\sin\frac{\beta}{2}\cdot\cos\frac{\gamma}{2}$
b.3	$\cos\alpha+\cos\beta+\cos\gamma=4\sin\frac{\alpha}{2}\cdot\sin\frac{\beta}{2}\cdot\sin\frac{\gamma}{2}+1$
b.4	$\cos\alpha+\cos\beta-\cos\gamma=4\cos\frac{\alpha}{2}\cdot\cos\frac{\beta}{2}\cdot\sin\frac{\gamma}{2}-1$
b.5	$\sin 2\alpha+\sin 2\beta+\sin 2\gamma=4\sin\alpha\cdot\sin\beta\cdot\sin\gamma$
b.6	$\sin 2\alpha+\sin 2\beta-\sin 2\gamma=4\cos\alpha\cdot\cos\beta\cdot\sin\gamma$
b.7	$\cos 2\alpha+\cos 2\beta+\cos 2\gamma=-4\cos\alpha\cdot\cos\beta\cdot\cos\gamma-1$
b.8	$\cos 2\alpha+\cos 2\beta-\cos 2\gamma=-4\sin\alpha\cdot\sin\beta\cdot\cos\gamma+1$
b.9	$\sin 3\alpha+\sin 3\beta+\sin 3\gamma=-4\cdot\cos\frac{3\alpha}{2}\cdot\cos\frac{3\beta}{2}\cdot\cos\frac{3\gamma}{2}$
b.10	$\sin 4\alpha+\sin 4\beta+\sin 4\gamma=-4\sin 2\alpha\cdot\sin 2\beta\cdot\sin 2\gamma$
b.11	$\sin 4\alpha+\sin 4\beta-\sin 4\gamma=-4\sin 2\alpha\cdot\sin 2\beta\cdot\cos 2\gamma$
b.12	$\cos 4\alpha+\cos 4\beta+\cos 4\gamma=4\cos 2\alpha\cdot\cos 2\beta\cdot\cos 2\gamma-1$
b.13	$\cos 4\alpha+\cos 4\beta-\cos 4\gamma=4\sin 2\alpha\cdot\sin 2\beta\cdot\cos 2\gamma+1$
b.14	$\sin 5\alpha+\sin 5\beta+\sin 5\gamma=4\cos\frac{5\alpha}{2}\cdot\cos\frac{5\beta}{2}\cdot\cos\frac{5\gamma}{2}$
b.15	$\sin 5\alpha+\sin 5\beta-\sin 5\gamma=4\sin\frac{5\alpha}{2}\cdot\sin\frac{5\beta}{2}\cdot\cos\frac{5\gamma}{2}$
b.16	$\cos 5\alpha+\cos 5\beta+\cos 5\gamma=4\sin\frac{5\alpha}{2}\cdot\sin\frac{5\beta}{2}\cdot\sin\frac{5\gamma}{2}+1$
b.17	$\cos 5\alpha+\cos 5\beta-\cos 5\gamma=4\cos\frac{5\alpha}{2}\cdot\cos\frac{5\beta}{2}\cdot\sin\frac{5\gamma}{2}-1$
b.18	$\sin 6\alpha+\sin 6\beta+\sin 6\gamma=4\cdot\sin 3\alpha\cdot\sin 3\beta\cdot\sin 3\gamma$

b.19	$\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2\cos\alpha\cdot\cos\beta\cdot\cos\gamma+2$
b.20	$\sin^2\alpha+\sin^2\beta-\sin^2\gamma=2\sin\alpha\cdot\sin\beta\cdot\cos\gamma$
b.21	$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=-2\cos\alpha\cdot\cos\beta\cdot\cos\gamma+1$
b.22	$\cos^2\alpha+\cos^2\beta-\cos^2\gamma=-2\sin\alpha\cdot\sin\beta\cdot\cos\gamma+1$
b.23	$\sin^2\frac{\alpha}{2}+\sin^2\frac{\beta}{2}+\sin^2\frac{\gamma}{2}=-2\sin\frac{\alpha}{2}\cdot\sin\frac{\beta}{2}\cdot\sin\frac{\gamma}{2}+1$
b.24	$\sin^2\frac{\alpha}{2}+\sin^2\frac{\beta}{2}-\sin^2\frac{\gamma}{2}=-2\cos\frac{\alpha}{2}\cdot\cos\frac{\beta}{2}\cdot\sin\frac{\gamma}{2}+1$
b.25	$\cos^2\frac{\alpha}{2}+\cos^2\frac{\beta}{2}+\cos^2\frac{\gamma}{2}=2\sin\frac{\alpha}{2}\cdot\sin\frac{\beta}{2}\cdot\sin\frac{\gamma}{2}+2$
b.26	$\cos^2\frac{\alpha}{2}+\cos^2\frac{\beta}{2}-\cos^2\frac{\gamma}{2}=2\cos\frac{\alpha}{2}\cdot\cos\frac{\beta}{2}\cdot\sin\frac{\gamma}{2}$
b.27	$(\sin\alpha+\cos\alpha)(\sin\beta+\cos\beta)(\sin\gamma+\cos\gamma)=2(\sin\alpha\sin\beta\sin\gamma+\cos\alpha\cos\beta\cos\gamma)+1$
b.28	$\frac{\cot\alpha}{\sin\beta\cdot\sin\gamma}+\frac{\cot\beta}{\sin\gamma\cdot\sin\alpha}+\frac{\cot\gamma}{\sin\alpha\cdot\sin\beta}=2$
b.29	$\tan\alpha+\tan\beta+\tan\gamma=\tan\alpha\cdot\tan\beta\cdot\tan\gamma$
b.30	$\tan n\alpha+\tan n\beta+\tan n\gamma=\tan n\alpha\cdot\tan n\beta\cdot\tan n\gamma$
b.31	$\tan\alpha\cdot\tan\beta+\tan\beta\cdot\tan\gamma+\tan\gamma\cdot\tan\alpha=\sec\alpha\cdot\sec\beta\cdot\sec\gamma+1$
b.32	$\cot n\alpha\cdot\cot n\beta+\cot n\beta\cdot\cot n\gamma+\cot n\gamma\cdot\cot n\alpha=1$
b.33	$\cot\alpha+\cot\beta+\cot\gamma=\cot\alpha\cdot\cot\beta\cdot\cot\gamma+\csc\alpha\cdot\csc\beta\cdot\csc\gamma$
b.34	$\frac{\cot\alpha+\cot\beta}{\tan\alpha+\tan\beta}+\frac{\cot\beta+\cot\gamma}{\tan\beta+\tan\gamma}+\frac{\cot\gamma+\cot\alpha}{\tan\gamma+\tan\alpha}=1$
b.35	$\cot\frac{\alpha}{2}+\cot\frac{\beta}{2}+\cot\frac{\gamma}{2}=\cot\frac{\alpha}{2}\cdot\cot\frac{\beta}{2}\cdot\cot\frac{\gamma}{2}$
b.36	$\cot\frac{\alpha}{2}=\frac{\sin\beta+\sin\gamma}{\cos\beta+\cos\gamma}$
b.37	$\left(1+\tan\frac{\alpha}{4}\right)\left(1+\tan\frac{\beta}{4}\right)\left(1+\tan\frac{\gamma}{4}\right)=2\tan\frac{\alpha}{4}\cdot\tan\frac{\beta}{4}\cdot\tan\frac{\gamma}{4}+2$
b.38	$\frac{\tan\alpha\cdot\tan\beta-1}{\cos^2\gamma}+\frac{\tan\beta\cdot\tan\gamma-1}{\cos^2\alpha}+\frac{\tan\gamma\cdot\tan\alpha-1}{\cos^2\beta}=\frac{3}{\cos\alpha\cdot\cos\beta\cdot\cos\gamma}$
b.39	$\sin\left(\alpha+\frac{\beta}{2}\right)+\sin\left(\beta+\frac{\gamma}{2}\right)+\sin\left(\gamma+\frac{\alpha}{2}\right)=4\cos\frac{\alpha-\beta}{4}\cdot\cos\frac{\beta-\gamma}{4}\cdot\cos\frac{\gamma-\alpha}{4}-1$
b.40	$\sin^3\alpha\cdot\cos(\beta-\gamma)+\sin^3\beta\cdot\cos(\gamma-\alpha)+\sin^3\gamma\cdot\cos(\alpha-\beta)=3\sin\alpha\cdot\sin\beta\cdot\sin\gamma$

Fig.2 – Basic geometric-trigonometric equalities

Activities and experimentation

As a first activity, the students practised recognising if a specific expression is an immediate consequence of one of those illustrated in the figure (Fig.2). The following figure (Fig.3) presents one of the cards used for the exercise.

Proposal n. 1	
w.01	$\dfrac{\cos\alpha+\cos\beta+\cos\gamma-1}{4}=\sin\dfrac{\alpha}{2}\cdot\sin\dfrac{\beta}{2}\cdot\sin\dfrac{\gamma}{2}$
w.02	$\cot(\alpha-\beta)=\dfrac{1+\tan\alpha\cdot\tan\beta}{\tan\alpha-\tan\beta}$
w.03	$\cos^4\alpha-\sin^4\alpha=\cos^2\alpha-\sin^2\alpha$
w.04	$\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\cdot\tan\beta\cdot\tan\gamma=0$
w.05	$\dfrac{1-\tan\alpha}{1+\tan\alpha}=\dfrac{\cot\alpha-1}{\cot\alpha+1}$
w.06	$\dfrac{\sin(\alpha-\beta)}{\sin\alpha\cdot\sin\beta}=\cot\beta-\cot\alpha$
w.07	$\tan\alpha+\cot\alpha=\dfrac{2}{\sin 2\alpha}$
w.08	$\sin 2\alpha+\sin 2\beta+\sin 2\gamma=3\sin\alpha\sin\beta\sin\gamma+1$
w.09	$\sin^2 2\alpha+\sin^2 2\beta+\sin^2 2\gamma+2\cos 2\alpha\cdot\cos 2\beta\cdot\cos 2\gamma=2$
w.10	$\dfrac{1-\tan\alpha\cdot\tan 2\alpha}{1+\tan\alpha\cdot\tan 2\alpha}=\dfrac{\cos 3\alpha}{\cos\alpha}$
Identify any expressions that are consequences of one of the forty basic geometric-trigonometric equalities. Indicate below their ID.	
Answers:	

Fig. 3 – recognition questionnaire

Subsequently, students were offered some cards with problems to verify their ability to recognise the expressions that are constructed by combining two or more geometrical equalities between the 40 of the figure (Fig. 2). The following figure (Fig. 4) presents one of the boards used for the exercise.

Each student had 15 minutes to answer the question.

Proposal n. 2	
x.01	$4\cos\alpha\cdot\cos\beta\cdot\cos\gamma=-\cos 2\alpha+\cos 2\beta+\cos 2\gamma-1$
x.02	$\sin\alpha+\sin\beta=2\cos\dfrac{\gamma}{2}\left(\cos\dfrac{\alpha}{2}\cdot\cos\dfrac{\beta}{2}+\sin\dfrac{\alpha}{2}\cdot\sin\dfrac{\beta}{2}\right)$

x.03	$\dfrac{\tan\alpha\cdot\tan\beta+\tan\beta\cdot\tan\gamma+\tan\gamma\cdot\tan\alpha-1}{\sec\alpha\cdot\sec\beta}=\sec\gamma$
x.04	$\dfrac{\cos\alpha}{\sin\alpha\cdot\sin\beta\cdot\sin\gamma}+\dfrac{\cos\beta}{\sin\beta\cdot\sin\gamma\cdot\sin\alpha}+\dfrac{\cos\gamma}{\sin\gamma\cdot\sin\alpha\cdot\sin\beta}-2=0$
x.05	$\cos4\alpha+\cos4\beta-4\sin2\alpha\cdot\sin2\beta\cdot\cos2\gamma=\cos4\gamma+1$
Identify any expressions that are a consequence of two or more of the forty expressions provided. Indicate below their ID.	
Answers:	

Fig. 4 – Combination of basic geometric equalities

Finally, the students were asked to demonstrate one of the five expressions proposed using their knowledge and the 40 equalities (see Fig. 2). The following figure (Fig.5) presents one of the boards used for group exercise.

Each group had 20 minutes to answer the question.

Survey n. 3	
y.1	$\dfrac{\sin2\alpha+\sin2\beta-2\sin\gamma\cos\gamma}{4\cos\alpha}=\cos\beta\cdot\sin\gamma$
y.2	$\cos^2\alpha-\cos^2\gamma=\sin^2\beta-2\sin\alpha\cdot\sin\beta\cdot\cos\gamma$
y.3	$\dfrac{\tan\alpha+\tan\beta+\tan\gamma}{\tan\alpha\cdot\tan\beta\cdot\tan\gamma}=\sin^2\alpha+\cos^2\alpha$
y.4	$\cos\dfrac{\alpha}{2}=\dfrac{\sin\beta+\sin\gamma}{\cos\beta+\cos\gamma}\cdot\sin\dfrac{\alpha}{2}$
y.5	$\sin4\alpha+\sin4\beta=-8\sin2\alpha\cdot\sin\beta\cdot\cos\beta\cdot\cos2\gamma+\cos4\gamma\cdot\tan4\gamma$
Solve, in groups, one of the following geometric equalities. Basic geometric-trigonometric equalities can use without demonstration.	
Resolution:	

Fig. 5 – Resolution, in groups, geometric equalities

Finally, the figure (Fig. 6) presents one of the worksheets used, at the end of the path, to monitor the skills acquired (acquired and previous) individually.

Each student has had 45 minutes to answer the question.

Proposal n. 4	
z.1	$2\cos^2\dfrac{\alpha}{2}+2\cos^2\dfrac{\beta}{2}+2\cos^2\dfrac{\gamma}{2}=\cos\alpha+\cos\beta+\cos\gamma+3$
z.2	$2\sin4\gamma=4\sin2\alpha\cdot\sin2\beta\left(\cos2\gamma-\sin2\gamma\right)$

z.3 $\cos\frac{\alpha}{2}+\cos\frac{\beta}{2}+\cos\frac{\gamma}{2}=4\cos\frac{180°-\alpha}{4}\cdot\cos\frac{180°-\beta}{4}\cdot\cos\frac{180°-\gamma}{4}$ Solve, individually, one of the following geometric-trigonometric equalities. Basic geometric-trigonometric equalities can use without demonstration.
Resolution:

Fig. 6 – Individually final test. Solution of geometric equalities

Discussion and results

21 students L11 and L12 year of schooling participated in this research. Of these, 12 were males and nine females. During the course topics and insights were discussed that are generally not carried out at school during the period of study in Italy.

The first figure (Fig.7) presents the percentages of the exact answers achieved by the students during the first exercise.

85.71% of students have identified that question w.01 is an immediate consequence of b.3 of fundamental geometric equalities. This question had a correct answer from 91.67% of the 12 male students, which corresponds to 11 students and 77.78% of the nine female students corresponding to 7 people.

76.19% identified that question w.04 is a consequence of equality b.29. This question was answered correctly by 83.33% of males, which corresponds to 10 students and 66.67% of female students who correspond to 6 people.

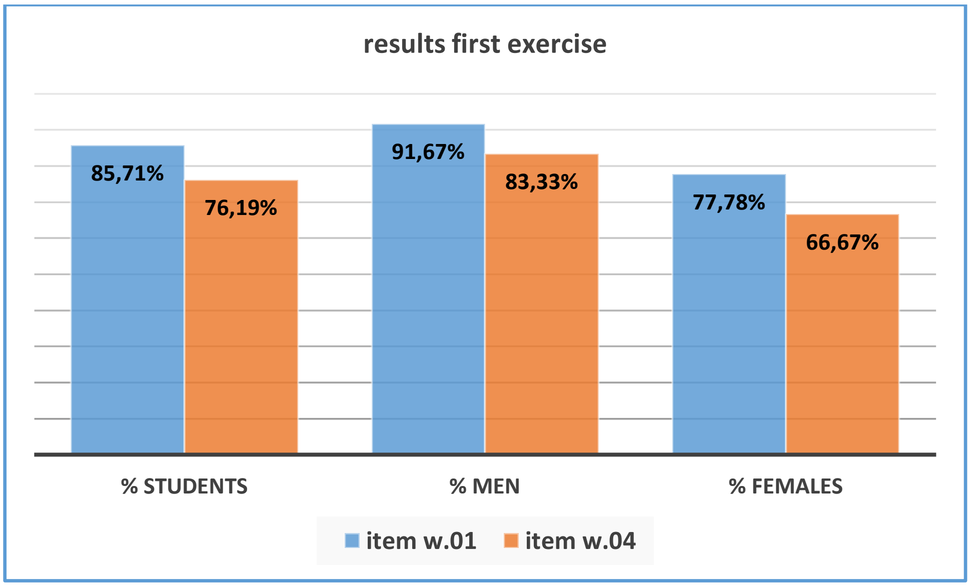

Fig. 7 – Results first exercise

The percentage of responses cannot be considered a good result. In fact, the task was only to observe and decide to use only the four fundamental operations of arithmetic. Also, the results of the percentages of correct answers of the other cards were similar and included in a range of ±1.82% compared to the values presented in the graph.

The figure (Fig.8) shows the percentages of the correct and non-correct answers for the second exercise.

The exercise asked to identify the equalities that would result in combinations of two or more basic expressions.

9.52% of the students have identified that question x.1 is a consequence of the equality b.7 (see Fig.2). Only two students answer incorrectly.

In fact, the item is an equivalent formulation of the equality b.7.

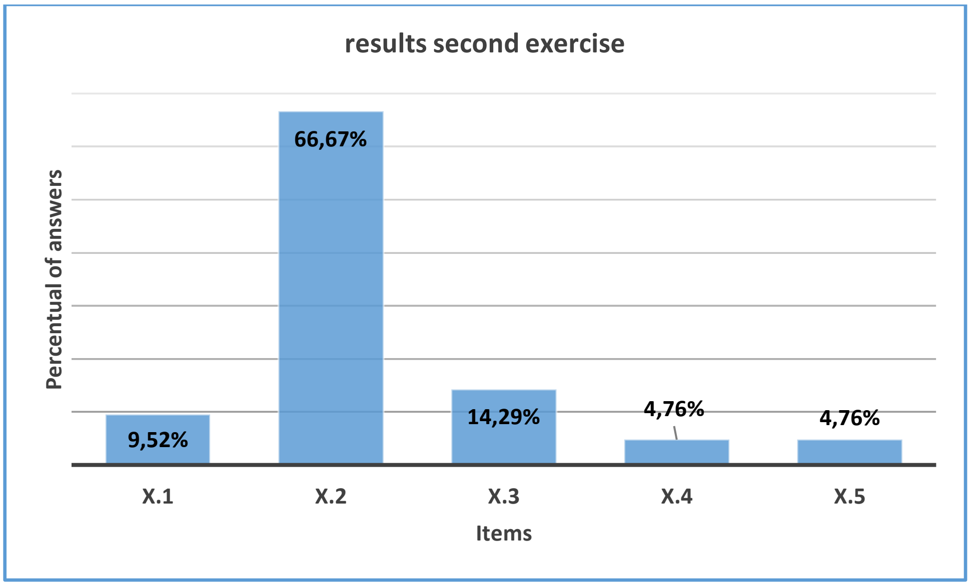

Fig. 8 – Results second exercise

66.67% of the students indicated that the question x.2 is a consequence of the questions b.1 and b.2. These (14) students answered correctly. In fact, the equality can achieve by following simple algebraic operations using items b.1 and b.2.

14.29% of the students indicated that the question x.3 is a consequence of the question b.31. These three students answered incorrectly. In fact, item x.3 is a writing equivalent of equality b.31.

4.76% of the students identified that question x.4 is a consequence of question b.28. These one student answered incorrectly. In fact, the element x.4 is identical to the question b.28.

4.76% of the 21 students identified that question x.5 is a consequence of question b.13. These one student answered incorrectly. Indeed, equality x.5 is identical to question b.13.

We can think, after reading these results, that even the students of the second phase of experimentation like those of the first phase, cannot read carefully and consequently understand the deliveries of the tasks to be performed. Our conjecture could further deepen (with subsequent research) extending the statistical sample to other Italian students of school age L11 and L12. The exercise asked to identify the equalities that would result in combinations of two or more basic expressions.

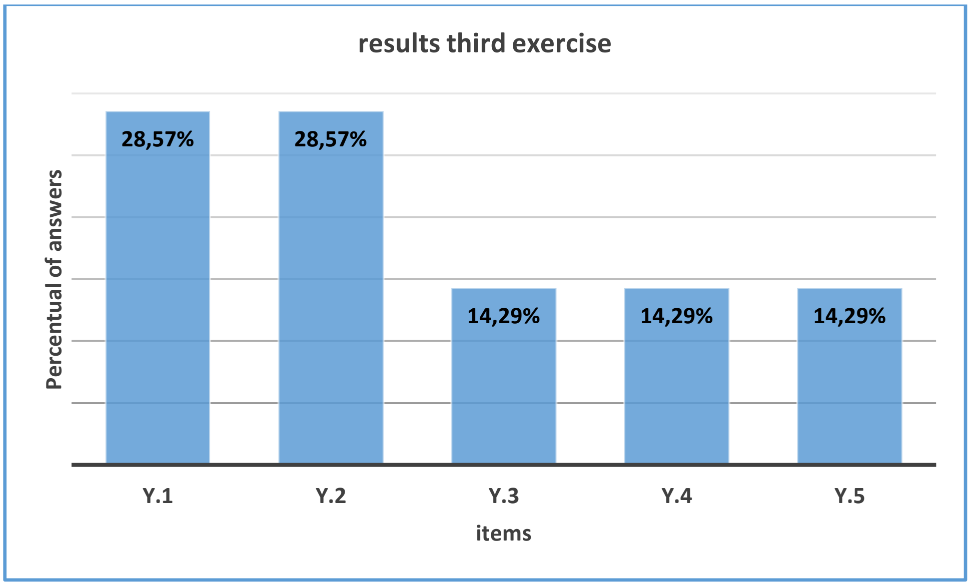

Fig. 9 – Results third exercise

Figure (figure 9) presents the percentages of the seven groups of students who have solved a specific item of the third exercise.

28.57% of the groups, which corresponds to 2 groups of the seven formats, solved the question y.1. The question presents a medium difficulty.

28.57% which corresponds to 2 groups, solved the question y.2. The question presents a medium difficulty.

14.29% (corresponds to 1 group) solved the question y.3. The question presents a medium-high difficulty.

The 14.29% that corresponds to 1 group solved the question y.4. The question presents a medium difficulty.

14.29% which corresponds to 1 group, solved the question y.5. The question has a medium difficulty.

It is important to stress that all the groups have solved one of the five proposed questions successfully.

As can be seen in the last figure (Fig. 10):

42.86% of the students, which corresponds to 9 people, solved the question z.1. The question does not present a high level of difficulty. The largest number of students chose this question.

33.33% of the students, which corresponds to 7 people, solved the question z.2. The application does not present a high level of difficulty.

The 19.05% that corresponds to 4 students solved the question z.3. The question presents a higher level of difficulty with than the other two proposed.

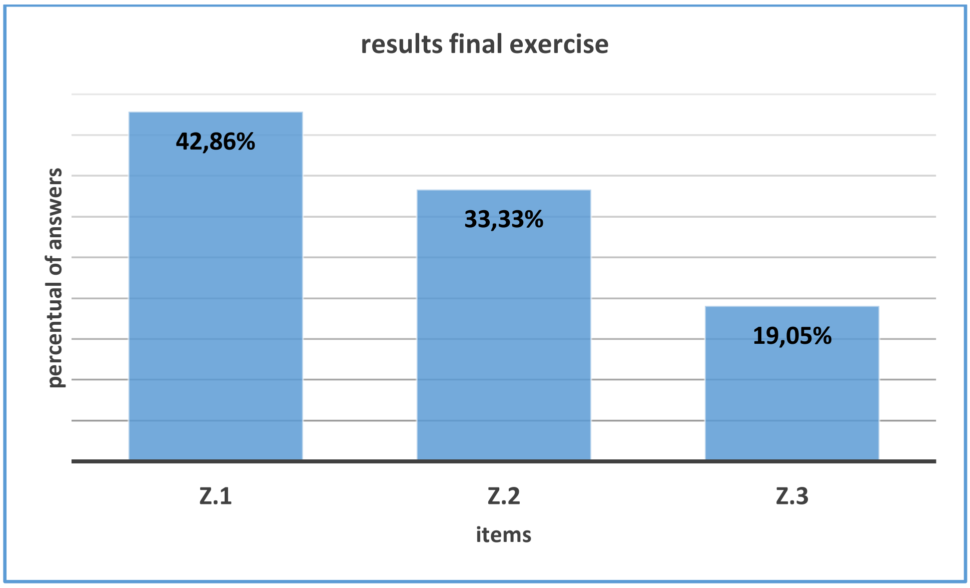

Fig. 10 – Results final exercise

Finally, it should note that the 4.76% that corresponds to 1 student, has not been able to completely or correctly solve any item of those proposed.

Also in the second phase, we did not intervene with the students to understand the motivations of the decision-making process that guided their response choices.

Conclusion

The authors intend to continue the research following the same basic methodology. This new experiment that will involve a new group of students will have three phases. The first phase will concern only linear elements. The second phase will involve only angular elements. The third phase will involve both linear elements and angular elements. In this way - by gradually inserting linear elements, angular elements and linear and angular elements - the information field provided is expanded and the requests to the students become more complex. This way of proceeding seems to the authors to evaluate the degree of ability of the students in managing much information on increasingly complex problematic situations close to the world of abstraction compared to the real world in which they live every day.

None of the students in the group highlighted particular heuristic skills.

Declaration of Conflicting Interests

The author declared that they had no conflicts of interest concerning their authorship or the publication of this article.

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