

Comparing fractions at the age of 11 through the use of the zone of proximal development

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Abstract. *A fundamental concept associated with the sociocultural theory of learning by Vygotsky (1967, 1978, 1987 and 1998), the zone of proximal development is employed in our present article to help an 11-year old student overcome his difficulties when comparing basic mathematical concepts such as fractions.*

Key words. *Sociocultural Theory Vygotsky, collaboration, Chalkin.*

Sommario. *Nell'articolo si descrive come un concetto fondamentale associato con la teoria socio-culturale dell'apprendimento - zona di sviluppo prossimale di Vygotskij 1967, 1978, 1987 e 1998 - viene impiegato per aiutare uno studente di 11 anni a superare le sue difficoltà quando gli si chiede il confronto con i concetti matematici di base come per esempio le frazioni.*

Parole chiave. *Sociocultural Theory Vygotsky, collaboration, Chalkin.*

Introduction

A fundamental concept associated with the sociocultural theory of learning by Vygotsky (1967, 1978, 1987 and 1998), the zone of proximal development, abbreviated as *zpd*, is employed in our present article to help an 11-year old student overcome his difficulties when comparing basic mathematical concepts such as fractions.

The whole culture of the article is inspired by the work done by Chalkin (2003) to view the concept above as deeply as possible and the new dimension he provides the word collaboration. Not just an urge to move forward but more any communication with a more experienced person that finally adheres to the fundamental principle of sociocultural theory (Vygotsky). Learning takes place first in the intra level, among human beings, and then it is appropriated to oneself, the inter level according to Vygostky. Chalkin's effort in approximating this concept is really ingenious since it combines not only the austerity by sciences that are based on such a strict style such as mathematics, but also the verbal interpretation of the zone of proximal development in a more loose sense that can present attributes of this concept that remained entirely unheeded or latent, through a time, during which the sociocultural theory of learning has extended its clout on a lot of disciplines.

The main core of this paper is a detailed interview with an 11-year old student and his insight

on the fundamental concept of fraction. By employing a question-response mechanism, we focus on a human triangle, the student, the interviewer and the mother, and beyond that an invisible world of demeanors and convictions toward Mathematics. A rather limited learning ambient is extended on sociocultural principles as presented by Vygotsky and refined and fortified by Chalking, especially regarding the notion of collaboration. Our next paragraph comes more natural to expounding what about this whole paper is.

Methodology

We proceeded in this article by the method of a direct interview with an 11-year old primary school student on the island of Rhodes, who is trying to master the idea of the fraction especially when compared to unity, something very important and connected to the nature of understanding what a fraction means. The theoretical background that fosters our work is the zone of proximal development, defined by Vygotsky and exemplified in a unique way by Chalkin.

Our student under observation is a bilingual student that by nature is more liberated to unveil language mechanisms required and employed in the process of learning, since what the native speaker sometimes takes for granted is a bone of contention for the bilingual one, as a result of an unavoidable conflict between the mother tongue and the prevalent environmental language. In the bilingual learning environment the zone of proximal development can be naturally enriched by the intervention of a parent, usually user of a language different than that of the instruction, and the difference in culture, not only in the general sense, but also in the restricted microsociety of the school that this parent or more knowledgeable person was exposed to as a student.

We intend to discover through our student's words how this zone works and to what extent language and cultural factors intervene to shape and redefine the concept that has occupied valuable time of whole generations of contemporary teachers, the zpd.

Main article

In the following article T will stand for the interviewer, the teacher and S the interviewee, the student. Our student seems to have some obstacles in understanding how a fraction is compared to unity and define the right sign of inequality he has to complete in his respective homework that depicts the level of understanding in this direction.

By taking on the role of the more knowledgeable person, as defined in the zone of proximal development, we do serve a double purpose: on one side we help the student deepen into the fundamental mathematical notion of the fraction and on the other side reevaluate the importance of this zone to the benefit of improving our teaching activity and the learning of mathematics itself.

T When you compare fractions to unity, you seem to encounter a real problem.

S Yes, I do. Most of the classmates face it. We discuss it with each other and correct the wrong ones

(already a vestige of the microculture society of the classroom that forms a modicum of social ambient so that the Vygotskian social learning process takes place).

T But how about the trick I gave you? Can you recite it?

S Sure. When the numerator is less than the denominator, we put the angle at the 1 and its nose

at the fraction.

Language seems to be a dynamic component in our lesson. Peter is not capable of translating into the proper terms “greater than”, “less than”. Instead, he develops his vocabulary to depict this situation. Sort of geometrical language, an angle and its nose, he does not even mention the word vertex. By adopting the zone of proximal development so nicely articulated by Vygotsky, “the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (Vygotsky, 1978, p. 86, Chalkin, 2003, p.40), we try to give the path of Peter’s mind from the pure thought to the word that expresses it. The geometric shape of an angle with its nose forms the path, capable of leading Peter from the thought, the mnemonic rule comparing the numerator to its respective denominator, to the word “greater than”, “less than”. It is undeniably subjective how this path, meaning, is formed on every mind, regardless of the similarities and differences that may show up by direct comparison, but the ultimate goal, the appropriation of the concept remains the same.

T The comment I made is not contained in the book. Do you think this is a problem? Would it help if it was stated?

S Mrs M had already mentioned it

T But you seem to have completely forgotten it. When I mentioned that to you, the expression on your face changed.

It was really a reassuring glance that motivates all sociocultural proponents to stick even closer to that theory.

S You said that to me

T But the teacher also said it to you and the rest of your classmates I suppose. Didn’t she?

S Yes, but when you said it to me, it was the solution. I thought it was a rule

T It does make a difference if the teacher addresses you personally, the way we talk face to face with each other.

S Sure, I even told my mother that

Peter is a bilingual student. When it comes to mathematics, he needs some help from a person in proximity, an application of the zone of proximal development. The criteria for such a decision is not only the greater competence of the second person, as stated by Vygotsky, but also the emotional security provided by a family member. His mother is a permanent refuge. But here, we face one of the primary obstacles again while learning in a bilingual learning ambient, the conflict between the mother tongue and the environmental language. We believe that this dispute can be proven quite fruitful in the context of the zone of proximal development, by accepting multicultural approaches and bilateral loans from either language that might lead to the betterment of the mathematical performance. The Vygotskian zone might be enriched by the mixing of different cultural approaches, which undeniably pass through the language mechanism, and extended to one or more additional persons forming a tripartite group dedicated to deepening into the mechanism under which mathematical understanding is obtained. A phrase or a word that is not used in one language might trigger the interest of learners, native speakers of the others, and vice versa. All this can happen at an age that the child’s mental development is fundamentally shaped avoiding our coming back years ago at the high school or even at the college level to

rectify misconceptions, or better behaviors toward mathematics, whose hostility have such a negative impact on our daily lesson.

But let us return to our dialogue

T Did your mother mention something similar to the rule I gave to you?

S (thinking and smiling) Maybe. But you know, my mother calls the denominator the upper part. She does not know the word denominator in Greek.

T You can teach her

S It does not work, she forgets, and if pushed, she speaks Albanian

T That means she cannot avoid using words for her native language.

S Not often. When she gets frustrated, and we cannot solve the problem, she does that. And then she tells me her story from the Albanian primary school she attended, what the teacher at that time used to say, how he graded them on the blackboard etc.

T So, finally you finally deflect from mathematics and are drifted into a story. Is that helpful? If she tends to narrate all she has been through at your age, would it not be a waste of time? Could you not call one of your classmates to discuss your encountered difficulties with?

S If I cannot find him/her? Mum is always here, and I do relish listening to her stories when she was herself a kid.

Comment: Chalkin is a perennial source of inspiration regarding the zone of proximal development. Next, to the three categories of development, he emphasizes on, we can add the availability property, i.e. the natural presence of a more knowledgeable one to interact with the less experienced and eager to learn. This availability is sure to the benefit of the potential development as viewed by Chalkin, articulated in a magnificent way “but merely an indication of the presence of certain maturing functions, which enables a situation that gives a potential for meaningful, interventive action, Chalkin, 2003, p.43”. That indicator of the maturity process is given vent due to the sentimental proximity that a relative can offer, always being there, no matter how well prepared she is for helping with the teaching subject. Moreover, within the family, there is already a distribution of roles regarding their offspring education, another indicator of how important parental involvement is the first formation of a learning attitude. It is the interplay between the assistive and potential development. Let me go on with the interview.

T You get a lesson and a break at the same time

S Sometimes I feel well when I prove to her that I was right about an exercise because she was unable to work out the right solution. I even told her your rule, about comparing fractions to unity.

T How she reacted?

S I think she said that in Albanian because it reminded her of her childhood. Then she told me, why didn't you tell me that at the very beginning?

T I caused a competition between you and her

S But I won, I knew it before she could ever learn it by heart

Comment: We do observe that this ingenious idea of the zpd can still be worked and expanded to encompass a lot of our learning experiments. The more knowledgeable person, in this case, the mother, resorts to her educational past, to another culture, another society with very different rules and laws. It is unavoidable to do this. In this way, another time, another culture passes per se in

the education of our cognizing subject, justifying Vygotsky once more in his allegation that learning is indeed a sociocultural process.

T Peter, if you can answer me the last question, I will be thrilled. But I can give you a whole week to elaborate on that if you cannot provide me with an immediate answer. Do you see any advantage in our discussion with you? Did I put something else next to the teacher, your mother, everyone who might have helped you so far?

S You gave me the rule, now I do not make mistakes if a fraction is greater than or less of unity (let us remember the moment transition from the descriptive terms angle and nose to the austere mathematical terms “less than” once the rule, according to Peter, was provided).

T But even the teacher, your mother knew the answer in Albanian. What makes the difference that you pay more attention to me?

S You said it to me

T I am a teacher too

S You are not giving me grades unless that one on the scratch paper.

Comment: The undeniable presence of the evaluation presence and its repercussions on the learning itself. We live in a judgmental society, and we keep talking about continuous assessment as a means to improve our modern society. Should we conceptualize more about this process, maybe categorize it between immediate and latent evaluation, the way I interviewed Peter, and try to figure out the pros and cons between the combination of these two kinds for the sake of learning itself?

T Could I also add that I am not supposed to reprimand you for anything outside Mathematics the way that a typical teacher does?

S Let alone that.

Conclusions

The new educational model is comprising of an extension of the so-called zone of proximal development. The ideal would be a teacher-parent-additional teacher. The teacher represents curricula, national standards and policies and is embedded into a legislative, educational frame. The parent is free of procedural and pedagogical norms but inclined to support her offspring with any means available. She is a bearer of another generation under the clout of another educational school of thought, and in the case of a bilingual student, exposed to a different language mechanism. The third person, another teacher, would neutralize the effects of the above two. She is neither sentimentally involved nor a part of the evaluation process. Evaluation cultivates competition, but if we follow Peter’s answers, it can inhibit the under no restrictions law of learning, by taking the liberty of using the term “laws of learning”. Would the cooperation among all three be beneficial to the improvement of Peter’s performance and shed more light on the cognition process? It is a minimal sociocultural setting comprising discipline-hierarchy (teacher), sentiment-familiarity (parent) and liberty of expression (the additional teacher). This presupposes a well-organized educational system with no downsizing in the number of hired teachers, nation and statewide, as well as the provision of motivation for the accomplishment of such goals. Any school district, provided the availability of means, can materialize these tripartite meetings, to the benefit of the education itself.

Individualized learning: The paragon of a teaching ambient, but practically unattainable. Why not apply the common mathematical idea of the approximation to this target? We can dedicate office hours, equally-distributed to all interested students to make them feel they deserve and at the same time get a chance to a privileged learning setting.

The zone of proximal development is an equivalence relation in its pure mathematical sense. Vygotsky never exercised, while living the job of a mathematics teacher, but he undoubtedly possessed a mathematical thinking enviable by contemporary mathematicians. Another proof that language and mathematics are in their structure so close related. The zone by itself is minimal, let me not say minimum, learning ambient, where even a rudimentary form of social interaction takes place for the sake of learning. It is reflexive by definition. It is undeniably symmetric, given the center of this zone if the Vygotskian definition implies such a center. It is not only to the benefit of the less experienced one but also to the advantage of the more knowledgeable one, which encompasses the principles of modern didactics that in the learning process all involving sides are active. The students learn by talking with us, and we do improve our performance by coping through a dialogue with their understanding obstacles in the field of mathematics. This reciprocity defines the symmetrical property of the zone of proximal development. Finally, we can sense the last component of transitivity, as different zones interact with each other. By accomplishing collaboration, as stated by Chalkin.

The term ‘collaboration’ should not be understood as a joint, coordinated effort to move forward, where the more expert partner is always providing support at the moments where maturing functions are inadequate. Rather it appears that this term is being used to refer to any situation in which a child is being offered some interaction with another person about a problem to be solved, (Chalkin, 2003, p.54)

between two different zones with a human overlapping, either the less experienced or, the more experienced one, the benefits are somehow transferred as if the new participants had already taken place in the zones they did not personally take part in. By finishing the concluding paragraph, we can use even the last comment as a means to propagate mathematics among people who are by nature theoretically oriented. A notion that forms part of the contemporary science of psychology can be presented as an extraordinary but nevertheless example of a mathematical concept encountered in all curricula at schools and college that involve mathematics. The equivalence relation. By an imitation process, we can start figuring out similar situations where the so specialized mathematical vocabulary materializes in subjects that mathematics are traditionally absent if an adage like this can finally be supported about the interconnection of Sciences and the applicability of the sociocultural theory.

Declaration of Conflicting Interests

The authors declared that they had no conflicts of interest on their authorship or the publication of this article.

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