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article

La comunicazione didattica in Matematica: Aspetti critici ed esperienze di apprendimento

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Abstract. *Students' difficulties in learning mathematics are long-faced, intricate matters. We can face up to this problem through different perspectives. A link between difficulties with mathematics and Specific Learning Disabilities requires well-defined methods and tools, so we would like to consider difficulties about communication of mathematics' notions. In this article, there is an introduction about the literary aspects of mathematics linked to psychological difficulties in learning. There's also a review about experiences of communication and didactic training with ICT.*

Key words. *Mathematics, communication, ICT, didactics.*

Sommario. *La difficoltà degli studenti nell'apprendimento di concetti matematici costituisce una questione annosa e complessa, affrontabile secondo molteplici prospettive. Tralasciando la "difficoltà in matematica" come espressione di un disturbo specifico d'apprendimento, la cui trattazione richiederebbe modalità e strumenti ben definiti, può essere utile riflettere sulla difficoltà di comunicare adeguatamente i contenuti della disciplina. Nell'articolo saranno esposti gli aspetti del linguaggio matematico che contribuiscono a determinare una predisposizione psicologica negativa verso l'apprendimento della disciplina stessa e sarà proposta una breve rassegna di esperienze di comunicazione efficace supportate dalla tecnologia nella didattica della matematica.*

Parole chiave. *Matematica, comunicazione, tecnologia, didattica.*

Introduzione

I dati OCSE PISA 2012, che hanno misurato le competenze degli studenti quindicenni in matematica e in problem solving, ci dicono che gli alunni italiani non conseguono buoni risultati in matematica. Le loro competenze matematiche, infatti, si situano significativamente al di sotto della media OCSE e, per di più, se confrontiamo i risultati ottenuti dalle ragazze e dai ragazzi, risulta che il differenziale medio in matematica tra i 30 paesi OCSE è pari a 11 punti a favore dei maschi, invece l'Italia è la quart'ultima tra questi paesi con un divario pari a 18 punti. L'OCSE sottolinea che tali scarsi risultati sono correlati con alcune idee e atteggiamenti diffusi, come il credere di saper risolvere i problemi di matematica (self-efficacy), l'autostima nelle proprie capacità matematiche (self-concept) e anche la notevole dose di ansia e di stress con cui si affronta la matematica. Il differenziale persiste anche a parità di istruzione dei genitori, di professione, di area geografica, di frequenza e di tipologia di scuola superiore. Pertanto, in Italia, al basso rendimento in matematica registrato per gli alunni quindicenni in generale, si associa il notevole gap esistente nei risultati riportati dalle ragazze e dai ragazzi. Alcuni studi hanno sottolineato come le metodologie di insegnamento della matematica siano rilevanti per abbassare il differenziale di genere e favorire lo sviluppo di *competenze matematiche*. A tale proposito, si richiamano due definizioni di competenza "…combinazione di conoscenze, abilità e [attitudini] atteggiamenti appropriati al contesto" (Raccomandazione del Parlamento Europeo e del Consiglio, 18 dicembre 2006) e "La competenza non si limita agli elementi cognitivi (che implicano l'utilizzo di teorie, concetti o conoscenze tacite), ma comprende anche aspetti funzionali (competenze tecniche), qualità interpersonali (per esempio, competenze sociali o organizzative) e valori etici" (Cedefop, 2004). L'apprendimento, quindi, non avviene acquisendo passivamente e successivamente le singole componenti contenutistiche, non avviene rapidamente, ma avviene in tempi lunghi e in un confronto continuo con il mondo reale, richiede un processo dove l'allievo deve sviluppare *abilità metacognitive e relazionali*, svolgere un ruolo attivo, diventando egli stesso protagonista del proprio apprendimento.

Nell'approcciarsi al tema della difficoltà in matematica può essere utile richiamare anche l'apparentemente ironica ma calzante diagnosi di "atteggiamento negativo", proposta da Rosetta Zan (2007). L'autrice, avendo raccolto 1600 temi autobiografici di alunni riguardanti le loro esperienze di apprendimento della matematica, ha elaborato una prospettiva caratterizzata da una multidimensionalità dell'atteggiamento, il quale risulta caratterizzato dalla dimensione emozionale, dal senso di auto-efficacia e dalla visione individuale riguardante la disciplina. La proposta di Rosetta Zan non intende soltanto rendere conto della complessità dell'atteggiamento verso la disciplina, ma desidera soprattutto dar spazio a un'interpretazione dell'esperienza in grado di indicare un percorso di miglioramento da seguire. La componente dell'esperienza didattica che attraversa, coinvolge e connette le dimensioni dell'atteggiamento è la comunicazione, punto di partenza del presente discorso. Per comprendere le modalità nelle quali le dinamiche comunicative si configurano come supporto od ostacolo all'esperienza didattica, è quindi necessario effettuare una loro analisi, specificamente riferita alla matematica.

L'insegnante che s'ispira a una visione della disciplina ridotta alle regole di applicazione del simbolismo convenzionale, tralasciando le potenzialità offerte da uno stile di comunicazione efficace, limita l'esperienza di apprendimento degli alunni e pone le basi per lo sviluppo di un atteggiamento negativo verso la matematica. Un simile evento può predisporsi nel caso in cui l'insieme di registri linguistici presenti nel contesto didattico generi una confusione tale da offuscare il linguaggio formale specifico della matematica, il quale, per la sua complessità, richiede attenzione e utilizzo sapiente e imprescindibile. È pertanto necessario gestire con cura e professionalità qualsiasi tentativo di coniugare il rigore proprio della didattica matematica con una comunicazione informale, sempre più diffusa in contesti istruttivi nel tentativo di migliorare la relazione docente-allievi.

La presente trattazione s'ispira a una visione dell'insegnante quale titolare di un duplice ruolo, consistente sia nella trasmissione di contenuti che nella guida dell'esperienza di apprendimento. Per tale ragione ci si sofferma sulla gestione della comunicazione didattica da parte del docente, sulle sue percezioni riferite alla disciplina e sulle sue caratteristiche personali in grado di agevolare l'esperienza di apprendimento. Gli studi finora condotti sull'argomento prevedono progetti di potenziamento della comunicazione didattica basati su gruppi di discussione *online* e utilizzo di altre risorse di rete. Un simile approccio è incoraggiato, nonché analizzato nelle sue potenzialità e margini di perfezionamento.

Apprendimento attivo della Matematica e Comunicazione

Già nel 1969, il pedagogista americano Edgar Dale, nel corso dei suoi studi sintetizzati mirabilmente nel famoso "*Cono dell'apprendimento*" (vedi Fig.1), evidenziava come la nostra memoria sia influenzata in modo significativo dalle esperienze e che tanto più quest'ultime implicano un coinvolgimento attivo tanto più perdurano nei nostri ricordi. Infatti, dalle analisi di Dale emerse che, dopo due settimane, tendiamo a ricordare solo il 20% di ciò che ascoltiamo ma il 70% di ciò che diciamo e il 90% di ciò che diciamo e facciamo. È evidente il ruolo importante che gioca la *Comunicazione* nei processi di apprendimento di tutte le discipline e, a maggior ragione, della matematica.

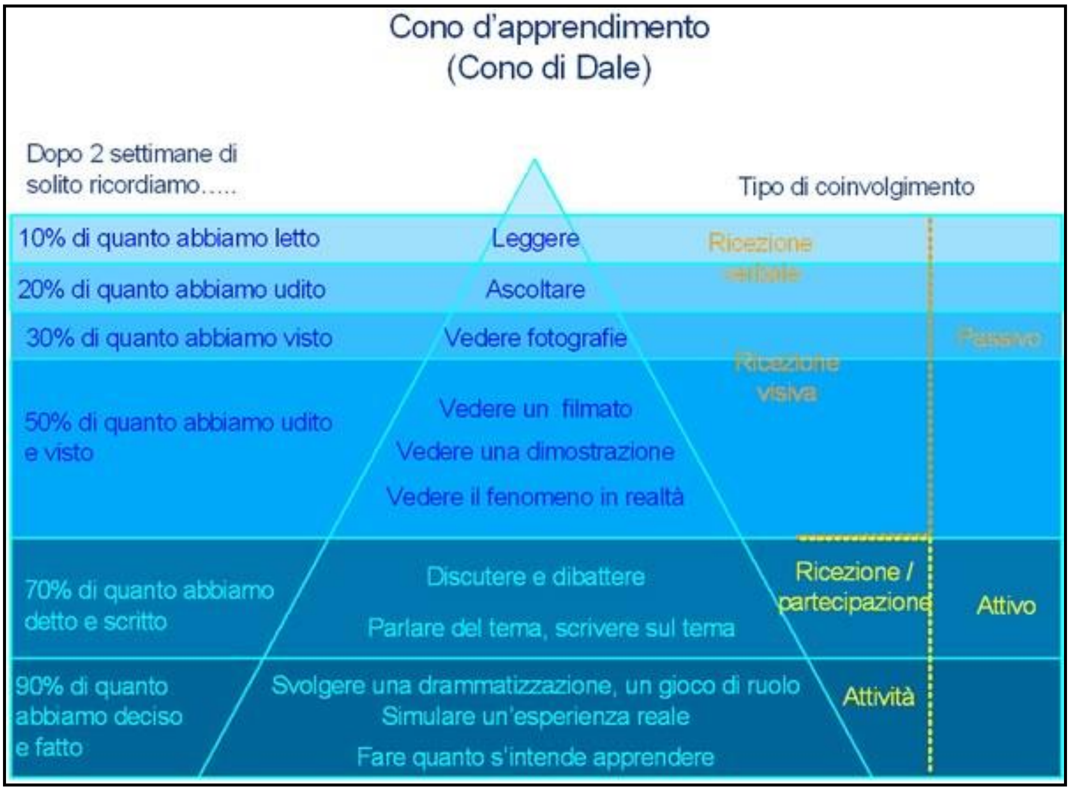

Fig.1 – Cono di apprendimento di Dale

Per convincersi di ciò, si può riprendere la ricerca condotta da Anna Sfard, descritta nel suo libro "*Psicologia del pensiero matematico. Il ruolo della comunicazione nello sviluppo cognitivo*" (2009), che «si rivolge allo studio del pensiero dell'essere umano in generale e del pensiero matematico in particolare», in cui si sostiene che la diffusa difficoltà nell'apprendere la matematica tragga origine dall'ambiguità insita nel nostro linguaggio. Infatti, è dichiarato: «quali sono le caratteristiche della matematica che la rendono così difficile da essere appresa» e, tra le cause, viene indicato il fatto che la disciplina ha alla base un substrato di regole logiche che la rendono sfuggente e inafferrabile. Quindi, la Sfard dedica il suo sforzo a dirimere la complessità che lega l'apprendimento e il pensiero creativo, *dando al linguaggio un ruolo costitutivo* e coniando il termine «*comognizione*» combinazione di comunicazione e cognizione.

Elementi formali e informali del linguaggio matematico

Nel tentativo di ricostruire l'evoluzione del linguaggio scientifico, Halliday (1993) aveva già individuato - come frutto di tale percorso - la tendenza a formulare i processi evitando l'aspetto verbale, per poterli quindi rendere in forma sostantivata. Questo implica un passaggio di considerazione da *processo* a *cosa*, al fine di rendere linguisticamente possibile la reciproca attribuzione di processi, di ordinare e predire gli eventi nel mondo fisico. La costruzione di gruppi nominali, pur fornendo un contributo in termini di categorizzazione della conoscenza teorica, metodologica e tecnologica, sembra aggiungere complessità al linguaggio scientifico. Schleppegrell (2007), poi, nell'analizzare la complessità del linguaggio matematico, individua ulteriori sfide d'interpretazione, consistenti nelle formulazioni multimodali dei concetti matematici - in termini di composizioni di parole e simboli - e nelle complesse strutture linguistiche presenti nel discorso matematico (del tipo: "*a è b, c è d, quindi e è f*"). La prospettiva emergente da tali studi è quella di *favorire un approccio multimodale al registro linguistico matematico*, in modo da risultare potenziata la funzione semiotica del linguaggio, a sostegno della componente simbolistica.

Un elemento di ulteriore complessità nella comunicazione dei contenuti afferenti alla disciplina è costituito dalla presenza di una molteplicità di registri linguistici presenti nel contesto didattico (D'Amore, Fandiño Pinilla, 2007), quali:

- Un linguaggio formale specifico della matematica;
- Un linguaggio dichiarativo orale dell'adulto che ha come oggetto la matematica;
- Un linguaggio dichiarativo scritto dell'adulto;
- Un linguaggio dichiarativo orale dell'allievo;
- Un linguaggio dichiarativo scritto dell'allievo;
- Un linguaggio di comunicazione, cioè dialogico, dell'adulto diretto all'allievo;
- Un linguaggio dialogico dell'allievo diretto all'adulto;
- Un linguaggio dialogico dell'allievo diretto a un suo pari.

L'insegnante, che ha già concettualizzato il contenuto che intende comunicare, può permettersi di cambiare anche continuamente il registro linguistico, ben consapevole che si tratta di rappresentazioni diverse dello stesso concetto. L'allievo, invece, proprio in quanto tale, non ha a disposizione il concetto che l'insegnante vuole trasmettergli, ma soltanto le rappresentazioni e spesso finisce con il confondere il concetto astratto con le sue rappresentazioni concrete. Di fronte a un tale coacervo di rappresentazioni, la difficoltà del ruolo formativo risiede nell'imporre il *rigore* che la didattica matematica esige.

La necessità di rigore può generare ulteriori importanti difficoltà nella comunicazione della matematica, poiché i destinatari di tale comunicazione sono persone prive di un particolare *background* disciplinare (Dedò, 2012). Un'esperienza italiana ha dimostrato la possibilità di mantenere il rigore richiesto dalla didattica matematica pur assumendo uno stile di comunicazione – per quanto possibile – informale. Le ricerche del Centro Interuniversitario di Ricerca per la Comunicazione e l'Apprendimento Informale della Matematica, per l'appunto, hanno suscitato un vivo dibattito, in quanto alcuni matematici hanno sostenuto che l'utilizzo dell'aggettivo "informale" riferito alla disciplina, basata notoriamente sul formalismo, dia luogo ad una evidente contraddizione. I fondatori del Centro hanno, però, motivato la loro scelta e i loro intenti, sostenendo la presenza di una grande domanda e un gran bisogno di comunicazione informale della matematica, sia nelle scuole che - più in generale - nella società.

Intuitivamente, il linguaggio "informale" ha una caratteristica che mal si adegua al rigore matematico, consistente nella sua potenziale ambiguità. Il concetto di ambiguità, trovando però un sinonimo favorevole nell'idea di molteplicità, può essere degno di considerazione positiva. Secondo i fondatori del Centro, infatti, la molteplicità dà luogo ad arricchimento, perché consente di esplorare regioni inaccessibili per chi non è avvezzo a un linguaggio formale. L'arricchimento derivante dal *concetto di ambiguità* consiste anche nella facilitazione dell'attività associativa, strumento fondamentale per costruire e trasmettere pensieri, immagini e concetti astratti. Si può obiettare a tutto ciò, sostenendo che un approccio informale alla matematica costituisca un rischio. L'informalità, ad ogni modo, non esclude il controllo sulle possibili interpretazioni dei contenuti comunicati. Un utile strumento di controllo in ambito didattico è costituito dall'ascolto attivo (Gordon & Bruch, 1974; Rogers & Farson, 1979), consistente nell'accogliere incondizionatamente l'informazione di ritorno della comunicazione inoltrata, valorizzandola e integrandola al fine di migliorare la pratica. Il controllo va potenziato se l'approccio informale prevede immagini (animazioni virtuali o modelli tridimensionali che possano essere toccati e manipolati) per comunicare un contenuto matematico, dato che non si tratta di rappresentazioni perfettamente corrispondenti al concetto da trasmettere. In seguito sono stati presentati studi a supporto del valore didattico dell'uso di immagini e contenuti virtuali nel suggerire concetti, ragion per cui la loro presenza in un approccio informale alla matematica va sicuramente favorita.

Difficoltà nella comunicazione didattica della matematica

Nel riportare le loro esperienze d'apprendimento in matematica, gli studenti sottolineano spesso un eccessivo focus del docente sui concetti piuttosto che sulle modalità linguistiche di trasmissione degli stessi. Si potrebbe rigettare tale critica, sostenendo che la matematica dia luogo a un linguaggio universale con una struttura costituita da definizione, teorema e prova (Jamison, 2000) e che i simboli corrispondano alle parole del linguaggio umano, avendo la funzione di trasferire significato (Bezuk et al., 2003). Una tale considerazione, seppur valida, non autorizza a progettare un'esperienza didattica fondata sull'idea secondo la quale nella presentazione di un teorema è insita la dimensione relativa alle competenze di comunicazione nel linguaggio matematico, le quali competenze comunicative sarebbero quindi trasmesse in automatico. *L'insegnante di matematica ha, quindi, il dovere di favorire lo sviluppo di competenze di comunicazione matematica, indipendentemente dalla trasmissione dei concetti di riferimento* (Kabael, 2012); si tratta di trasformare il sapere in un "sapere da insegnare" (D'Amore, 1999b).

Gray (2004) fornisce un'ipotesi sul perché gli insegnanti tendano a trascurare le abilità comunicative, facendo riferimento alla teoria dell'auto-efficacia di Bandura (1997). L'autore, infatti, sostiene che l'insegnante tenda a trascurare la componente comunicativa perché non conosce adeguate modalità linguistiche di trasmissione del concetto, oppure perché ritiene di non essere in grado di comunicare efficacemente, esibendo in tal caso deficitari livelli di auto-efficacia. La letteratura sull'argomento propone uno strumento di valutazione di auto-efficacia del docente (Language of Mathematics Teacher Efficacy Scale; Gray, 2004), utile per comprendere le origini della lacuna nella funzione d'insegnamento e poter eventualmente intervenire, in modo tale da compensare il deficit. Kabael (2012), in una sperimentazione condotta sugli insegnanti di matematica di livello corrispondente alla scuola media italiana, ha riscontrato difficoltà nella modalità di affrontare un discorso di valutazione e negazione di alcune proposizioni. È, quindi, possibile ipotizzare che l'alunno, rapportandosi a un docente con simili caratteristiche, possa assimilare le modalità comunicative proposte, *contribuendo alla diffusione di un modello di apprendimento-insegnamento basato su una <comunicazione inefficace>*.

Esperienze di comunicazione efficace supportate dalla tecnologia

Dalgarno e Colgan (2007) hanno favorito un esperienza di sviluppo professionale per insegnanti di matematica, predisponendo loro un ambiente di discussione disciplinare online. Dalle analisi dei *self-report* dei partecipanti è emerso che un simile contesto di apprendimento si rivela utile per favorire la proattività nella ricerca di esperienze di sviluppo, nonché per incentivare la ricerca di risorse didattiche di qualità. La *community* di riferimento era ispirata ai principi della comunità di pratica (Lave e Wenger, 1991), essendo organizzata in modo tale da favorire la condivisione delle risorse, la collaborazione tra utenti e l'integrazione di nuovi membri.

L'uso delle tecnologie di informazione e comunicazione incoraggia, quindi, l'apprendimento attivo e collaborativo, nonché la conoscenza individuale e la struttura dell'insegnamento frontale. Brahim et al. (2014), in un'indagine condotta tra gli insegnanti, hanno però rilevato che *gran parte degli insegnanti di matematica utilizza le risorse della rete Internet a scopi personali e non didattici*. Partendo da tale constatazione, risulta necessario proporre percorsi di formazione all'uso delle tecnologie di comunicazione e informazione, in modalità funzionale al perfezionamento professionale e allo sviluppo di ambienti di insegnamento-apprendimento attivo.

Le tecnologie di informazione e comunicazione costituiscono non soltanto una modalità utile allo sviluppo e all'aggiornamento professionale. Molti docenti, dopo averne appreso i criteri di utilizzo ottimali e aver ottenuto una prospettiva delle risorse fruibili, si servono di supporti multimediali anche per le loro attività di insegnamento. In particolare, Khorasani (2012) ha verificato l'utilità dell'attività di insegnamento della matematica col supporto di collegamenti a pagine web. I risultati di apprendimento sono stati soddisfacenti e l'autore li imputa alla fruibilità e riutilizzabilità del materiale, ipotizzando anche che una spiegazione mediata possa risultare più chiara, al netto delle interferenze proprie della comunicazione faccia a faccia. Quest'ultima sembra essere l'idea alla base della diffusione di esperienze di *blended learning* associati a corsi scolastici e universitari di matematica e altre discipline scientifiche. Inoltre, D'Aprile (2011) ha evidenziato nelle sue ricerche che gli strumenti forniti dalle piattaforme o ambienti virtuali di supporto aiutano l'organizzazione dei documenti, dei riferimenti bibliografici, delle attività didattiche, semplificano e favoriscono la compilazione di quiz, l'assegnazione e la correzione di compiti e, soprattutto, attenuano alcune delle difficoltà di comunicazione tra docente e allievi.

L'accostamento della pedagogia dell'apprendimento tra pari all'insegnamento della matematica ha dato luogo a una nuova prospettiva. Non è recente, infatti, la volontà di introdurre – in ambito didattico – principi di apprendimento collaborativo, al fine di favorire e potenziare attività di pensiero, ragionamento e metacognizione (Lucangeli, Coi e Bosco, 1997). Un percorso matematico prevede la costruzione di assunti matematici, lo sviluppo e la valutazione di argomenti, la selezione e l'applicazione di diversi tipi di rappresentazione (NCTM, 1989). L'attività metacognitiva prevede, invece, sulla base di concetti matematici, l'abilità di spiegare i processi di ragionamento sottostanti (Mevarech e Kramarski, 1997), nonché la generale riflessione sui processi individuali di controllo e autoregolazione (Schoenfeld, 1985).

Da tempo si sostiene che l'interazione con l'altro possa facilitare i suddetti processi (Piaget, 1926; Vygotskij, 1978); è invece più recente l'idea che la tecnologia possa aiutare a elevare cognitivamente il ragionamento, supportando le attività di pianificazione, monitoraggio e anticipazione di possibili risultati (Zembat, 2008). Il *peer tutoring* favorisce la dimensione motivazionale all'apprendimento. Da qui il progetto di creare ambienti virtuali di apprendimento tra pari. I risultati di simili esperienze supportano l'idea di base: *la partecipazione a discorsi matematici in un contesto di comunità di pratica favorisce la trasmissione di idee e strategie* (De Corte, 2000; Sfard, 2000). La *membership*, inoltre, potenzia il concetto di sé e, quindi, una positiva attitudine verso l'apprendimento anche dei concetti più ostici della matematica (Ma, 1997). Tsuei (2012), in una rassegna sul tema, ha tracciato le linee da seguire nell'implementazione di *peer tutoring system di didattica della matematica: è necessario istituire un ambiente virtuale sincrono, improntato alla costruzione collaborativa della conoscenza* (Scardamalia e Bereiter, 1994), *per favorire l'intervento e la partecipazione, necessari perché si realizzi quell'apprendimento attivo, in grado di perdurare più a lungo nella memoria (Cono dell'apprendimento, Edgar Dale)*.

Conclusione

L'introduzione di supporti tecnologici (LIM, software 3D, risorse di rete, ecc.) alla didattica rappresenta attualmente, nella maggior parte dei casi, ancora una semplice prospettiva, poiché si può registrare più che altro una diffusa alfabetizzazione informatica e l'uso della LIM come strumento di visualizzazione in grande. La digitalizzazione di processi nelle attività scolastiche è ancor oggi agli albori. Quindi, sarà necessario attendere i dovuti tempi di maturazione socio-culturali per poter usufruire a pieno dei vantaggi tecnologici applicati alla didattica.

I risultati degli studi proposti dalla letteratura sull'argomento offrono promesse incoraggianti e spunti di notevole interesse. È possibile affermare che, allo stato attuale, le esperienze e le proposte letterarie tracciano "Linee Guida" alle quali ispirarsi, nel caso in cui si riscontrino dinamiche di comunicazione inefficace, da voler risolvere seguendo metodologie di insegnamento che utilizzano le prospettive d'innovazione tecnologica.

A tale proposito, non si può fare a meno di segnalare l'importante esperienza in atto in alcune scuole italiane che aderiscono ad *"Avanguardie Educative",* un movimento di *innovazione educativa*, nato dall'iniziativa congiunta di INDIRE con un primo gruppo di scuole, che porta a sistema le esperienze didattiche più significative della scuola italiana.

Il movimento offre e alimenta una "galleria di idee" che sono sperimentate dalle scuole aderenti e presentate quali esperienze come in un mosaico, il cui obiettivo comune è rivoluzionare il *modo di "fare scuola" nei tempi, nello spazio, nella didattica.*

Sicuramente *Avanguardie educative* rappresenta un tentativo di riunire in modo organico le diverse *best practices* italiane nel campo dell'innovazione didattico-tecnologica.

Si spera che questo e altri contributi successivi possano contribuire a definire in modo ottimale le modalità di supporto alla didattica in matematica, proponendo percorsi concreti e metodologicamente fondati, utili a sviluppare contemporaneamente concetti matematici e competenze comunicative, affinché realmente la Matematica diventi un patrimonio di tutti.

Contributo degli Autori

Tutti gli autori hanno contribuito all'ideazione dello studio. Valentina Mastrogiacomo ha curato i primi quattro paragrafi. Erminia Paradiso ha curato i rimanenti due paragrafi, e tutti gli autori hanno contribuito alla scrittura del manoscritto.

Dichiarazione di conflitti di interesse

Gli autori dichiarano di non avere conflitti di interesse rispetto la paternità o la pubblicazione di questo articolo.

Deposito dei materiali dell'attività

Al seguente link sono depositati eventuali materiali inerenti questo l'articolo. Questi materiali nel tempo potranno essere modificati e arricchiti seguendo l'evoluzione delle idee sottostanti o/e future sperimentazioni svolte dall'autore dell'articolo.

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Bibliografia/Sitografia

Avanguardie educative. http://avanguardieeducative.indire.it/

Bandura, A. (1997). *Self-efficacy: The exercise of control*. New York: Freeman.

Bezuk, N. S., Pothier, Y. M., & Cathcart, W. G. (2000). *Learning mathematics in elementary and middle schools.* Prentice Hall.

Brahim, N., Mohamed, B., Abdelwahed, N., Ahmed, L., Radouane, K., Khalid, S., & Mohammed, T. (2014). The use of the Internet in Moroccan high schools mathematics teaching: state and perspectives. *Procedia-Social and Behavioral Sciences*, 116, 5175-5179.

Cedefop, (2004) http://europass.cedefop.europa.eu/it/education-and-training-glossary

D'Aprile, M. (2011). "Blended learning" per studenti universitari di Matematica. *TD Tecnologie Didattiche*, 19(3), 198-202.

Dalgarno, N., & Colgan, L. (2007). Supporting novice elementary mathematics teachers' induction in professional communities and providing innovative forms of pedagogical content knowledge development through information and communication technology. *Teaching and teacher education*, 23(7), 1051-1065.

Dale, E. (1946) *Audio-visual methods in teaching.* New York: The Dryden Press.

D'Amore, B. (1999b). *Elementi di didattica della matematica*. Edizioni Pitagora.

D’Amore, B., & Pinilla, M. I. F. (2007). *Le didattiche disciplinari*. Edizioni Erickson.

De Corte, E. (2000). Marrying theory building and the improvement of school practice: A permanent challenge for instructional psychology. *Learning and instruction*, 10(3), 249-266.

Dedò, M. (2012). Rigour in Communicating Maths: A Mathematical Feature or an Unnecessary Pedantry?. In *Raising Public Awareness of Mathematics* (pp. 339-358). Springer Berlin Heidelberg. Elementary and middle schools. (3th Ed.) River, N.J: Merrill/Prentice Hall.

Gordon, T., & Bruch, N. (1974). *Teacher effectiveness training*. New York: PH Wyden.

Gray, V. D. (2004). *The Language of Mathematics: A Functional Definition and the Development of an Instrument to Measure Teacher Perceived Self-efficacy*, Ph.D. diss., Oregon State University.

Halliday, M. A. (1993). Towards a language-based theory of learning. *Linguistics and education*, 5(2), 93-116.

Jamison, R. E. (2000). Learning the Language of Mathematics. *Language and Learning Across the Disciplines*. 4, 45-54.

Kabael, T. (2012). Graduate Student Middle School Mathematics Teachers’ Communication Abilities in the Language of Mathematics. *Procedia-Social and Behavioral Sciences*, 55, 809-815.

Khorasani, M. K. (2012). An online approach to teaching mathematic formula through introducing web-page links. *Procedia-Social and Behavioral Sciences*,46, 3546-3550.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge University Press.

Lucangeli, D., Coi, G., & Bosco, P. (1997). Metacognitive awareness in good and poor math problem solvers. *Learning Disabilities Research & Practice,* 12, 219-244.

Ma, X. (1997). Reciprocal relationships between attitudes toward mathematics and achievement in mathematics. *Journal of Educational Research*, 90, 221–229.

Mevarech, Z. R., & Kramarsky, B. (1997). From verbal descriptions to graphic representations: Stability and change in students' alternative conceptions. *Educational Studies in Mathematics*, 32(3), 229-263.

Piaget, J. (1926). *The Language and Thought of the Child*. New York: Harcourt Brace & Company.

Raccomandazione del Parlamento Europeo e del Consiglio, 18 dicembre 2006.

Rapporto Nazionale OCSE PISA 2012, a cura di INVALSI.

Rogers, C., & Farson, R. E. (1979). Active listening. *Organizational Psychology*, 168-180.

Scardamalia, M., & Bereiter, C. (1994). Computer support for knowledge-building communities. *The journal of the learning sciences*, 3(3), 265-283.

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly*, 23(2), 139-159.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Orlando, FL: Academic press.

Sfard, A. (2000). Symbolizing mathematical reality into being–or how mathematical discourse and mathematical objects create each other. *Symbolizing and communicating in mathematics classrooms: Perspectives on discourse, tools, and instructional design*, 37-98.

Sfard, A. (2009). *Psicologia del pensiero matematico. Il ruolo della comunicazione nello sviluppo cognitivo*, Edizioni Erickson, Trento.

Vygotskij, L. S. (1978). *Mind in society: the development of higher psychological process* (M. Cole, V John-Steiner, S. Scribner e E. Souberman, a cura di), Cambrigde, MA.

Tsuei, M. (2012). Using synchronous peer tutoring system to promote elementary students' learning in mathematics. *Computers & Education, 58(4),* 1171-1182.

Zan, R. (2007). *Difficoltà in matematica. Osservare, interpretare, intervenire.* Springer.

Zembat, I. O. (2008). Pre-service teachers' use of different types of mathematical reasoning in paper-and-pencil versus technology-supported environments. *International Journal of Mathematical Education in Science and Technology*, 39(2), 143-160.

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article

Ethics in school evaluation: assessment as an educational tool

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Abstract. *This article analyses the theme of evaluation in schools and its importance for the global growth of students.*

The process of evaluation is not only an obligation which the teacher must carry out but also an important instrument which guides and enriches their teaching. It has great ethical significance because the teacher, must use evaluation as a means to understand the students' strengths and weaknesses and consequently guide choices made to render their teaching fruitful.

In the first part of the article different types of evaluation are listed and the modern evaluation for competences is analysed in particular.

In this type of teaching the teacher is only a director of situations and a cultural mediator who evaluates how the students prepare themselves to face up to real life tasks.

European school policy has encouraged the objective of lessening the gap between school and the workplace of the real world.

Evaluation in mathematics has been the subject of much attention. Indeed, it is believed that there is an important correlation between poor results in this subject and their fear of being tested.

An initial survey on the fear of Maths and on school in general is proposed as solution to this.

Finally, attention is brought to the fact that in order to carry out an individual global evaluation which encourages growth, culturally, morally and relationally, teamwork on the part of the teachers is essential.

Sommario. *In questo articolo è stato analizzato il tema della valutazione scolastica e della sua importanza per l'evoluzione globale degli alunni.*

Il processo di valutazione non è solo un obbligo cui l'insegnante deve assolvere, ma un importante strumento di lavoro che arricchisce e indirizza la sua didattica. Essa ha anche un grande significato etico perché l'insegnate, libero da qualsiasi condizionamento interiore o esterno, deve usare la valutazione come un mezzo per comprendere gli alunni nei loro punti di forza e di debolezza e, di conseguenza, per indirizzare la sua azione didattica e renderla fruttuosa.

Nella prima parte dell'articolo, sono elencate le diverse tipologie di valutazione ed è analizzata, con particolare attenzione, la moderna valutazione per competenze.

In questo modello didattico, il docente è solo un regista di situazioni e un mediatore culturale che valuta come gli alunni si predispongono nell'affrontare compiti di realtà, le strategie cognitive che usano e i risultati che ottengono.

La politica scolastica europea, che mira a sanare la frattura tra il mondo della scuola e quello del lavoro, indirizza verso azioni didattico – educative di questo tipo.

In questo lavoro è stata posta molta attenzione anche sulla valutazione in matematica. Si ritiene, infatti, che ci sia un'importante correlazione fra gli scarsi risultati degli alunni in questa disciplina e la loro ansia da prova.
Si propone, a questo scopo, agli insegnanti di condurre indagini preliminari sulla paura della matematica e su quella della scuola, in generale. Il docente dovrebbe aver cura di interpretare i risultati ottenuti per di indirizzare la sua azione didattica e realizzare una valutazione più completa.
Infine, si pone l'accento sul fatto che, per effettuare una valutazione olistica, che costituisca un'utile strumento di crescita sia culturale che morale e relazionale degli studenti, sia indispensabile il lavoro d'equipe fra gli insegnanti.

Parole chiave. *Ethical evaluation, Life skills evaluation, evaluation taxonomy, Math's evaluation*

Introduction

In this article the author analyses the different forms of school evaluation with a focus on the evaluation of both Life skills and Mathematics. The evaluation is considered, not only as a teaching tool, but also as a valuable educational instrument, essential in the ethical training of students which will benefit the entire society.

Ethical meaning of evaluation

The verb *to evaluate* can be used in many different contexts: ethical, political, civic education ones. In all these cases, the Latin origin of the word means giving a value must not be forgotten.
The assignment of a value cannot occur, however, in an objective way; it always depends on the subjectivity and the social conventions. The analysis of some situations, in fact, changes greatly depending on the context in which they happen.
Homosexuality, for example, was for centuries considered a disease or a serious sin. Today is assessed a decent choice for some States, while for others it is an anomaly or even worse, a crime. The opinion regarding this theme also changes a lot from person to person, even inside the same social group.
In contemporary pedagogy and teaching, the concept of evaluation has undergone a profound change.
The evaluation loses, therefore, its value punitive and becomes a tool to facilitate the learning of pupils. (Zavalloni, 1967).
A great difference exists between expressing judgement on a piece of work and using the evaluation as a teaching instrument.
Evaluation can become an important moment of growth for teachers and students alike, through which they can understand their weak strong and weak points and if necessary make changes to strategies used.
Various types of evaluation exist which permit teachers to observe students in different moments over the course of their studies.
Each one of these, if used by the educator in a fair and careful way, is of the same importance.

If the teacher, as a professional who continually reflects, gives the evaluation the correct value they will take advantage of the different types of evaluation.
This will enable the students to change their point of view and see the teachers as allies and as a reference point for their personal and cultural growth.
Evaluation can also be a way in which to bring the students closer to society and the workplace. Evaluation of competences, for example, is of great social value, because it aims to evaluate the cultural, professional, personal and cognitive abilities which the students will have to use over the course of their lives.
Finally, it would seem to be common sense to use evaluation of mathematical competences as an instrument to bring students closer to the subject. Evaluation can be an occasion for students to confront their fears and difficulties in order to develop their personalities and widen their cultural knowledge.

For a full evaluation at school

There are different types of tests that students may be subjected: predictive tests, formative tests, summative tests, testing for certified, authentic testing (Life skills education).
To encourage the learning of the students have to create a learning environment in which they can build their knowledge and develop their skills by working in collaborative groups. Only through constructive relationship with peers, they can activate their *zone of proximal* development (Vygotsky, 1939).
We find the premises of these concepts in the Russian psychology and, particularly, in the jobs of Vygotsky (1939).
According to these theories, the learning is a social education, that happens through the social construction before and, subsequently, through the progressive transfer of the external social activity, through signs, to the inside control; an example of this trial is the learning of the language before is listened and included and, only in a second moment, acquired.
To optimize this trial is necessary that in class a positive climate is established. The teacher should hold a democratic attitude, sincere, from positive leader, should be a point of available reference to the active listening and the help and should hold a director attitude of the activity, whose actors are the students. The teacher is an organizer of communicative situations for the learning and a cultural mediator. (Wraga, W.D.,1998).
Testing for certified means assessing achievements, but in reference to a macro context with the aim of acquiring a title that can be used in accordance with the law, as the degree, for example.The predictive tests are proposed at the beginning of the educational path, namely the beginning of a school year or a course of study. They are used to determine the starting level of the students and identifies possible misconceptions. It is an indispensable tool for the teacher in the planning of educational intervention.
The formative assestments are administered during the teaching activities and are designed to understand the level of understanding of the subject by students and to improve the work of the teacher.
The summative verifications are administered at the end of the topic and serve to measure what has been learned and to give a vote or a judgment that influences the student's career.
The objectives that are assessed are usually classified, by teachers, according to a hierarchical order: from the basic to the more advanced, to enable all pupils to start their work with serenity.

Anderson and Krathwhol (2001), for example, have taken as reference the previous taxonomy of Bloom, but amended it, giving more importance to the strategies of thought rather than the results.
According to them, goals that need to be investigated are six:

- *Remember*: that means recall information;
- *Understand*: explain idea or concept;
- *Apply*: use information;
- *Analyze*: distinguish between different aspects;
- *Evaluate*: justify own point of view;
- *Create*: creating own ideas.

The teacher who prepares the test must take care to prepare exercises that explore the objectives that have been set.

The Life skills teaching and its evaluation

It is usual for us, teachers and educators, in general to distinguish the pupils that have a good scholastic output from those that have a discreet output or poor straight, but they manifest only their abilities and their creativeness in the daily life.
Castoldi (2008) searches the cause in the typology of traditional teaching. He distinguishes among two macro models of teaching:

- The direct model or transmitted (“wall teaching”): in which the student is considered only an inactive knowledge fruit of the simplification of the reality;
- The indirect (“bridge teaching”) model: in which the pupil is in the center of the teaching and the role of the teacher is only that to create situations to hoc that really allows an effective and spendable learning the real world.

This new idea of teaching is on the center of the ultramodern Life skills teaching, of constructivism matrix.

The concept of skills is very complex and it has had a long evolution during the time, which will be discussed in the next section. (Castoldi, 2010).
In the last ten Fifteen years have taken, start as prosecution of the Cognitivism, the seams of pedagogic study of the constructivism and social Constructivism. According to these theories, the process of learning is effective only if it has as principal characteristic to be constructive, that is means “to reconstruct” what the subject already possesses: all the reception incentives from the outside enter the mind of the individual and they are laced to previous knowledge to build new knowledge.
The learning has, as construction, double nature:

- Relational: in relationship with the real world and with the social context;
- Dynamics: because it derives from the mobilization of previous knowledge.

Bruner (1986) speaks of a spiral learning that possesses three principal characteristics:

- Constructive;
- Social cultural: the social and cultural context is the theater where knowledge is produced;
- Situated: or tied up to the content and the context that produce it.

From these psycho - pedagogic theories, the concept of skill is born.

The first definitions of skill recall behaviorist vision, according to which it is identified with a performance of the observable and measurable subject. In practice, the concept of skill with coincided with the concept of ability.
In the following years, we assist to a progressive articulation of the concept that we can synthesize in three evolutionary directions (Castoldi, 2010):

- From the simple one to the complex: competence is considered as integration of the individual resources;
- From the outside to the inside: it affirms a progressive attention to the inside dimension of the not referable subject and not only to observable behaviors. In this optics, it places the Chomski's distinction among skill, as inside quality to the subject, and performance, agreement as observable behavior;
- From the abstract to the situated one: skill is appraised in base to the ability to face real assignments in specific social, cultural and operational contexts.

In effective way, Le Boterf (1994) reassumes the course that the concept of skill countersigned in "*the passage to know, to know how to be, to know how to act*", understood as synthesis among to know how to do and to know how to be, mobilized in operation of a problematic situation.

The Life skills and the International Organizations

Also the programmatic lines of the International Organizations push toward this Copernican revolution of the education that puts to the center the learning, understood as acquisition of Life skills.
Already in 1993, the WHO (World Health Organization) invited the formative agencies to make a formation able to equip the young people to face the difficulties in the life. These appeals are also received by the White Books of the European Commission. The second White Book (Cresson Flyun 1995), particularly, refers to one "cognitive society", that is endowed with that organized Knowledge essential to "stir in the world".
The "Key Competencies for a Successful Life and a Well-Functioning Society" final report of the DE.SE.CO search, 2003, defines the skills as a constellation and search the key - skills, valid for every sector of the knowledge. It substantially individualizes three Key skills:

- To interact in heterogeneous social groups;
- To autonomously act;
- To use tools in interactive way.

In the document "Recommendation 2006/962/CE of the European Parliament and of the Suggestion, of December 18th 2006, related to competences key for the permanent learning" [official Newspaper L 394 of the 30.12.2006, peg. 10] are individualized eight key–competences
The Key competences for the permanent learning they are a combination of knowledge, ability and appropriate attitudes to the context. Particularly, they are necessary for the personal realization and development, the active citizenship, the social inclusion and the occupation. The competence keys are essential in a society of the knowledge and they assure great flexibility to the workers to adapt in more rapid way to a world in continuous change and more interconnected. Besides, such competences are a factor of primary importance for the

innovation, the productivity and the competitiveness and they contribute to the motivation and the satisfaction of the workers and the quality of the job. The key skills must be acquired by:

- young people at the end of their cycle of compulsory education and formation, to prepare them to the adult life, above all to the working life, forming, at the same time, a base for the future learning;
- adults in the whole arc of their life, through a process of development and updating of their abilities.

The key competences for lifelong learning individualized are:

- The communication in the mother tongue;
- The communication in the foreign languages;
- The mathematical competence;
- The digital competence;
- Learning to learn;
- Social and civic competence;
- The sense of initiative and entrepreneurship;
- Cultural Awareness and expression.

What is the role of the teacher in the didactics for competences?

- He/She observes the class system;
- He/She designswith the colleagues the activities aimed to the acquisition of life skills. These activities must be challenging situations, drawn by the reality;
- He/She proposes the activities to the pupils and he observes them during the job, he/she annotates their reflections;
- Ago so that the pupils share with the classmates what they have learned and he drives this sharing, correcting possible errors, motivating and stimulating the students;
- He appraises the acquisition of the life skills. Before the beginning of the activities the teacher establishes the criterions of evaluation and words an index book valuation. The evaluation owes tend account, not only of the gotten results, but also of the motivation, of the approach to the situation problem and of the ability to work in group.

During the observation of classroom activities, the teacher compiles an evaluation rubric with which expresses the judgment as to the achievement of skills by pupils.

Table (see Table 1) is an example of an evaluation rubric in which it assesses the key skills: sense of initiative and entrepreneurship.

A table of this type can be used, such as for school projects that involve the local community: organization of events, festivals, markets, conferences, etc.

Table 1: Evaluation rubric

	Partial	Acceptable	Intermediate	Advance
Understanding of the task	He/she needs to be driven to gather the sense and the objectives of the projected action.	He/she has clear the general sense of the projected action and the essential objectives.	He/she has clear the sense and the objectives of the projected action.	He/ she has clear the sense and the objectives of the projected action and it is prefigured the route of the project.
Planning of the strategies	He/she labors to individualize the	He/she individualizes the	He/she individualizes the	He/she individualizes and it autonomously plans the

of action	fundamental steps of the route of the project and to direct the action.	fundamental steps of the route of the project.	steps of the route of the project and its own direct action and own operation in strategies of theirs relations.	steps of the route of the project and it directs their own action in operation strategies of their realization.
Control / regulation of the route of the project	He/she labors to hold under control the passages of the route of the project and to see the run again.	Is available and in degree of monitoring and to hold under control the essential passages of route of the project	He/she is available and in degree of monitoring and to hold under control the route project, re - directing, if necessary, the run.	He/she is available and in degree of monitoring and to hold under control the route of the project, verifying the coherence of it with the objectives and re - directing, if necessary, the run.
Social interaction	He/she has needed to be motivated and solicited to participate in the activities and to undertake in initiatives and common projects.	He/she participates in the activities but work to assume a purposeful role and to assume initiatives.	He/she participates in the activities with constructive attitude and spirit of initiative and availability to the search and the action.	He/she is motivated and it participates in the activities with constructive spirit, spirit of initiative and availability to the search and the action. And open to the dialogue, to the comparison and the sharing of knowledge, competences and resources for the realization of common projects.
Sensibility to the context	He/she must be stimulates to seek and to analyze useful information to the knowledge and the understanding of the context of action and to use her.	He/she search and it analyzes useful information to the knowledge and the understanding of the context of action and functional.	He/she search and He/he analyzes useful information to the knowledge and the understanding of the context of action using to define her objectives and lines of action.	He/she search and He/she analyzes critically useful information to the knowledge and the understanding of the context of action, using to define her objective meaningful and functional lines of action.

This variety of evaluation strategies and in particular formative assessment and life skills evaluation have high ethical and educational value. In fact, they allow students to increase their capacity of self-assessment that are indispensable for their real life. Therefore, these assessments can be rightly defined as assessments for learning (Black & Wiliam, 1998).

In practice, however, some empirical studies demonstrate that the evaluation practices used by teachers often are more discretionary, subjective and therefore unreliable (Poham, 1975; Domenici, 2007b; Webb & Jones, 2009).

Evaluation in Mathematics

The topic of assessment in mathematics has always been very popular among those who deal with it. It is thought that a poor evaluation further demotivates student's study of the subject,

which is very complex for the majority of students.
Anxiety about mathematics can have various origins (Ashcraft, Kirk E., Hopko, 1998)

- Objective difficulties of comprehension;
- Difficulties related to abstraction and categorization;
- Consignment not within the student's ability;
- Teacher's attitude;
- Excessive expectations on the part of the family;
- Limited self esteem.

This last aspect, in particular, pushes students to feel they are not up to the task. Students can thus believe that their results are not proportional to their efforts. This leads them to not complete with determination tests taken, to give random answers or even to hand in blank tests.
For this reason, it is possible a negative evaluation can hinder a student's motivation and trigger a vicious cycle: poor results – demotivation about the subject – lack of application to the subject.
Indeed, some studies have shown that anxiety about mathematics is not the simple consequence of cognitive difficulties, as it can also be identified in students without particular difficulties.
Before any school assessment, therefore, it would be very useful for teachers to analyze to what extent anxiety affects the performance of their students.
A deciding turnaround regarding this was the publication of the MARS (Mathematics Anxiety Rating Scale) questionnaire and the successive variations (Plake, Parker, 1982).
The original questionnaire contains 98 items, but has been adapted to different situations and age ranges.
Teachers can choose to use only some items or to create new ones, using those which are most significant to the task in hand for them. (from their point of view)
The tests require the student to consider certain situations connected to the teaching of mathematics and to express their state of agitation by way of a point system, 1-4.

The points obtained from rom each point are added in order to obtain a total score and to permit an evaluation.
The original 98 questions of the Mars questionnaire may be too many for a middle school or the first years of a high school. Furthermore they may discourage students with difficulties in reading or comprehension.

In line with the questionnaire for middle schools (Saccani e Cornoldi, 2005) the items could be classified in three groups:

- Anxiety regarding mathematical acquisition
- Anxiety regarding evaluation of mathematics
- Anxiety regarding school in general.

In the first group the following situations can be found:

- To follow an explanation of mathematics
- To buy a maths book
- To start a maths lesson
- To understand a graph
- To read a formula of geometry

- To understand the relationship between a formula and a physical phenomenon
- To apply a formula of geometry
- To calculate a percentage.

It is not difficult to find items to assess anxiety due the evaluation of mathematics:

- To have to do very difficult homework at home;
- The thought of a math's test set for the next day;
- Being asked to resolve an equation during an oral test;
- Being asked a geometry question during an oral test
- To take a math's exam at the end of the Middle school;
- To get the lowest mark of all the students in a test;
- To have to tell marks obtained in Maths to parents;
- To get 4/10 in a Maths test.

The third group of questions refer to anxiety connected to school in general. It is important to distinguish between the fear of mathematics and the distress that can be present in a school environment.

Examples of this could be:

- To answer end of unit geography questions;
- To read sheet music;
- To wait ones turn on the bench during a school football match;
- To play a song on a recorder;
- To prepare for a school trip;
- To ask a teacher for an explanation;
- To ask janitor for information;
- To deal with the presence in class of the strictest teacher.

In order for this type of survey to be functional, it must be carried out in such a way that the students can answer with serenity.

The teacher should explain that the questionnaire aims to better understand their difficulties and to find valid solutions together.

The questionnaire should be designed by the teacher to recreate possible situations from the daily routine of the students.

It would not make sense, for instance, to ask students from the first year of middle school if they are worried about solving an equation, due to the fact that this is usually dealt with in the third year. Likewise, to ask if a swimming competition might cause anxiety if the school does not actually take part in such events.

Care should be taken with the time at which the questionnaire is administered.

They should be spread out in order to let the students mentally recreate and temporarily relive each situation.

For the same reason, it is essential there is silence while it is being completed in order not to hinder concentration.

Soft background music could be played to create a relaxing atmosphere.

Every teacher must of course make the choices best suited to the class in question.

Particular care should be paid to the layout of the questionnaire. The questions must not be bunched together and the graphics should be captivating.

After data analysis has been done, the teacher could then create specific handouts which require each student to reflect on the situations and the reasons which cause the anxiety, on how much they may affect performance and productivity. Also strategies on how to deal with the problems should be considered.

A final stage could be a class discussion where the students are encouraged to share their own experiences and strategies used.

During the discussion, the students can come to understand that the fear of mathematics is an experience that they have in common, that they are understood by the teacher and classmates and receive advice regarding possible solutions.

The teacher may also decide to use a similar questionnaire at the end of the year in order to note any evolution that has taken place during the school year.

It is without doubt that this type of questionnaire is useful not only to make the students aware of their difficulties but also to guide the teacher in educational choices made.

As a reflective professional, the teacher must be flexible and adapt educational choices to the needs of the students, in order to minimize critical situations, to increase their motivation and their sense of self-efficacy.

This will lead to an ethically correct, precise and constant evaluation which takes into consideration not only targets reached but also the growth of the students as human beings.

The team work and its ethical role in the evaluation

For centuries, criteria that make the evaluation more objective are sought to increase its educational effectiveness. However, the personality and the educational choices of the teacher influence the evaluation of pupils highly.

The halo effect, for example, occurs when a teacher who appreciates a particular characteristic, for example the logic capacity, is influenced, often unconsciously, to evaluate other aspects and indicators.

The Pygmalion effect is another source of serious evaluation errors. The teacher, in fact, can have positive or negative prejudices about the abilities and commitment of the student. These biases affect the attitude of the teacher and this greatly affects the performance of the pupil triggering a vicious, sometimes virtuous, sometimes vicious, but always unethical.

Finally, the stereo effect, leads the teacher to think that the educational and cultural situation of the student cannot change. This false belief prevents the optimization of teaching and educational strategies (Booth,1978).

The only way to avoid these risks, offsetting the educational benefits of the evaluation, is the teamwork. The working group, in fact, allows a more complete view of the capacity, the knowledge and the skills of the pupil.

A professional team also undertakes to correct the errors of each of its members. This is done in a correct and productive way only if teachers establish correct relationships and create a serene environment of professional growth and sharing.

Conclusion

To this day it does not seem that evaluation is given the due importance it has for the growth of the individual and the progress of society.

In particular, the great ethical value of evaluation by teachers and its importance to society must be highlighted.
By evaluating teachers can help students to overcome their weaknesses, and place importance on their strong points whilst working with others with respect and enthusiasm.
Multiple forms of evaluation must be used in order to achieve these ambitious objectives.
Particular care must be paid to evaluation in mathematics, which has always been a subject met with trepidation by students.
Teachers cannot help but identify any existing fears in order that these become moments of reflection and growth.
The modern evaluation of competences would appear to be of particular use. To carry these out students must be confronted with real life difficulties.
By carrying out a real task, they are able to understand the importance of teamwork.
The group is not just a value for students but also for teachers, who by way of comparison with their work group, can grow professionally. By discussing the students in order to have a complete and objective vision they are able to evaluate with greater efficiency.

Declaration of Conflicting Interests

The authors declared that they had no conflicts of interest with respect to their authorship or the publication of this article.

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References

Anderson, L. W., Krathwohl, D.R. (2001). A Taxonomy for Learning, Teaching, and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives. New York, Longman.

Ashcraft, M.H., Kirk, E. P., Hopko D.R. (1998). On the cognitive conseguences of mathematics anxiety, in Donlan C. The development of mathematical skills, east Sussex, Psychology Press, pp 175-196.

Black, P., Wiliam, D. (1998). *Assessment and Classroom Learning. Assessment in Education*, 5(1), 7-71.

Booth, W.C. (1978). *Metaphor as Rhetoric: The Problem of Evaluation*, Critical Inquiry Vol. 5, No. 1, Special Issue on Metaphor pp. 49-72.

Bruner, J. (1986). *Actual Minds, Possible Worlds. Cambridge*, MA: Harvard University Press.

Castoldi, M. (2008). *Teaching wall and bridge*, the educator, n.1, '08/'09.

Castoldi, M. (2010). *Didattica Generale, Mondadori Università*, Milano.

Domenici, G. (2009). *Ragioni e strumenti della valutazione*. Napoli: Tecnodid.

Domenici, G., Chiappetta Cajola, L. (2005). *Organizzazione didattica e valutazione*. Roma: Monolite.

Domenici, G., Lucisano, P. (2011). *Valutazione, conoscenza, processi decisionali. Dibattito*. Educational, culture and psychological studies, 3, pp. 147-167.

Galliani, L., Bonazza, V., & Rizzo, U. (2011). *Progettare la valutazione educativa.* Lecce, Pensa Multimedia.

Le Boterf, G. (1994). *De the competence: Essai sur un attracteur étrange*, Paris, Les Editions d'Organisation.

Pellerey M., (2001). *The individual competences and the Portfolio*, Florence, You New Italia.

Plake, B.S., Parker, C.S. (1982). The development and validations of a revised version of Mathematics Anxiety Rating Scale (MARS), Educational and Pshycological Measurament, vol. 42, pp 551-557.

Resnick, L.B. (1995). *To Learn inside and out the school,* in C. Pontecorvo-A.M. Ajello-C. Zucchermaglio (edited by), the social contexts of the learning, Milan, LED, pp. 61-70.

Saccani, M., Cornoldi, C. (2005). Ansia per la matematica: la scala MARS-R per la valutazione e l'intervento metacognitivo, in Difficoltà in matematica, Erikson, Trento, pp. 133-152.

Wraga, W. G. (1998). *Democratic Leadership in the Classroom*: *Theory into Practice*. Presented at the Czech National Civic Education Conference Olomouc, Czech Republic, August 24 and 25, ERIC.

Zavalloni R. (1967). *"Valutare per educare",* La Scuola, Brescia.

"Recommendations related to the key competence", Recommendation 2006/962/CE of the European Parliament and of the Suggestion, of December 18 th 2006, related to competences key for the permanent learning" [official Gazette L 394 of the 30.12.2006, peg. 10]

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Developmental dyscalculia and the teaching of arithmetic

Lucia Maria Collerone

Abstract. *To understand Developmental Dyscalculia and organize an inclusive education that is functional to ensure the overcoming of the difficulties involved, it is necessary to know how the arithmetic is processed in the brain, both in subjects with neurotypical functioning and in those who, instead, have abnormal neural functioning that characterizes the dyscalculia disorder. From this information it is possible to address the choices of methodological practices for the classroom setting, following the directions of the research path named "Cervello, Cognizione & Educazione" which traces the basic steps to make possible a link between the Cognitive Sciences and the Educational Sciences. This article aims to describe what is numerical cognition, making a short review of the most recent researches concerning dyscalculia in the context of Cognitive Sciences and to create a possible methodological approach useful for teaching arithmetic basic competencies, in a school inclusion perspective.*

Key words. *Developmental Dyscalculia, learning arithmetic difficulties, methodology and didactics, neurobiology of dyscalculia.*

Sommario. *(La discalculia evolutiva e l'insegnamento dell'aritmetica). Per comprendere la discalculia e organizzare una didattica inclusiva che sia funzionale per garantire il superamento delle difficoltà che comporta, bisogna conoscere come l'aritmetica viene processata a livello cerebrale, sia in soggetti a funzionamento neurotipico che in chi, invece, ha un funzionamento anomalo che caratterizza il disturbo discalculico. Da tali informazioni è possibile trarre indicazioni d'indirizzo per la scelta di pratiche metodologiche per il contesto classe, seguendo le indicazioni del percorso di ricerca "Cervello, Cognizione & Educazione" che traccia i passi fondamentali per rendere possibile un collegamento tra le Scienze della Cognizione e le Scienze dell'Educazione. Questo articolo ha lo scopo di descrivere la cognizione numerica, dare una breve presentazione degli studi più recenti che riguardano la Discalculia Evolutiva nel contesto delle Scienze Cognitive e di tracciare un possibile percorso metodologico utile per la didattica dell'aritmetica di base, in una prospettiva di inclusione scolastica.*

Parole chiave. *Discalculia Evolutiva, difficoltà nell'apprendimento della aritmetica, metodologia e didattica, neurobiologia della discalculia.*

Introduction

From the 90s to today a path of international research has been concerned to establish a link between the Cognitive Sciences and the Educational Sciences with the ultimate goal of creating teaching methods that are "brain friendly", as they are based on the actual brain functioning in learning activities such as the basic skills, like those of numerical cognition. The methods created from this union are subjected to examination, through the method of the “research into action” conducted by the teachers in actual classroom activities, to establish if they are easily to be used in teaching and functional in their achievement of learning objectives that the institutional regulation demands.

The educational basis of those methods can grant an inclusive didactic approach because they are studied on “default learning brain mechanisms” and thus are able to facilitate learning in all the individuals that have a brain structure not affected by pathological diseases, that will be able to learn despite the individual and functional differences.

This research path based its knowledge on the information coming from Cognitive Sciences, which are interested in learning and brain functions both in subjects with typical or atypical functioning and in the results, on the different cognitive abilities, due to brain damage.

The information provided by the different research paths that characterize the Cognitive Sciences (neuroscience, psychology, linguistics, philosophy, artificial intelligence ...) allow to know what are the brain functions underlying learning and can become the basis for interventions that facilitate learning, by using methodological actions that are congruent to brain functioning or, also, they permit the creation of effective intervention methods for the recovery of lacking or not typical brain functioning.

In Italy the research path, called “Cervello, Cognizione & Educazione” led to the creation of a method for teaching reading, writing and calculation for the first grade of primary school, which was tested in research in action in the school years 2008/2010 in 32 first classes and is currently successfully implemented in many Italian schools.

This article aims to provide a definition of Developmental Dyscalculia that takes into account the latest and important researches in this area, providing information on the neurobiology of numerical cognition and a review of possible interventions that can be put in place in the choice of a “brain friendly” teaching method which ensures, through the use of congruent teaching actions with brain function, a learning innovative teaching approach of arithmetic, that is inclusive and effective because allows the learning of such basic skills of numerical cognition, despite of any not typical operations of the brain structure.

Developmental Dyscalculia (DD)

Arithmetic is very important in everyday activities and numerical cognition has a strong impact on education and even on occupational outcomes (Fuchs, 2009).

Developmental Dyscalculia (DD) is a learning disorder that limits the acquisition of arithmetic skills in a context of normal IQ. This difficulty in learning arithmetic at school level is common in about 5% of the pupils in primary schools and is a constant figure across the population in the countries where it has been studied.

An early sign of these difficulties can be observed in a delay of the acquisition of the pre-scholastic skills in the nursery and kindergarten years.

Dyscalculia is common both in girls and boys and children with this disorder can suffer severe emotional distress caused by their mathematical poor performance in school, which can lead to school phobia and mathematical anxiety (Marshall et al., 2006).

Commonly three subtypes of dyscalculia are considered:

- a specific isolated disorder of mathematics;

- a mathematic impairment in the context of general learning disabilities;
- dyscalculia with comorbid disorders (ADHD, dyslexia) occurs in approximately one quart cases (Kaufmann,2012) comorbidity with dyslexia seems to produce the most strong impairments.

C.M. Temple (1991, 1997) has given a definition of three kinds of DD that describes all the typical errors made by DD children:

1) “digit dyslexia” that is characterized by a difficulty in the acquisition of the lexical processes in the comprehension of the numerical system and in the calculation production.
2) “procedural dyscalculia” that is characterized by the difficulty in the acquisition of the procedures and the implicit algorithms in the calculation system;
3) “dyscalculia for numerical facts” that is characterized in the difficulty of acquisition of the numerical facts in the calculating system.

The most common behavioral characteristics of DD are:

- slow and error disposition learning and retrieval of mathematic facts from memory (Jordan and Montani, 1997);
- procedural deficit (immature calculation, problem solving, counting strategies) and delayed progress from finger counting to verbal counting (Geary, 2002);
- failed representation and access to numerical magnitude information like approximate and exact understanding of numerical quantities. (Butterworth, 1999);
- low accuracy during number comparison;
- poor performance on a range of basic numerical processing tasks including, verbal and dot counting, number comparison and number naming;
- low performance in approximate calculation in tasks that require manipulation of numerical magnitude representation (Jordan and Hanich, 2003);
- strong distance effect when it is required to compare symbolic or nonsymbolic quantities (Mussolin et al., 2010) or simple and double digit numbers (Ashkenazi et al., 2009b);
- impairments in spatial attention and spatial working memory (Ashkenazi et al. 2009a);
- basic deficit in numerical processing for number comparison and number naming.

Thanks to the help provided by these cognitive neuropsychological models it is possible to have a starting point for both diagnosis and for functional rehabilitation treatments or for the creation of effective teaching models able to offset early difficulties or to limit their effects of severity.

Neuro characteristics of Developmental Dyscalculia

In a neuroscientifical model the development of numerical processing elaborated by the brain is a neoplastic and maturational process, that, during the childhood and adolescence leads to the establishment of a complex and specialized network. At a few months of age this development begins with the ability to perform the basic skill of an approximate estimation of number for three or more items (Kaufmann, 2013).

When language appears children become able to symbolize the number linguistically using number words, they learn to count out and are able to perform simple arithmetical manipulation of quantities and numbers.

During the pre-school year and primary school children learn the Arabic number system, that is another way to symbolize numbers. In this period they learn also the Arabic place-value system and they develop a conceptual ability, the mental number line, that is a numero-spatial ability that enable them to operate with numerical symbols (Kucian et al., 2008).

The neural network that links multiple brain area is stimulated and made stronger with the increasing of practice and expertise.

Functional images have shown that multiple brain area are stimulated and different areas are active in correspondence of the different tasks:

- number words are elaborated in the left perisylvian region that are known to be the areas of speech;
- Arabic numerical representation is processed in the occipital lobe;
- basic numerical representations and numerical-spatial representation are processed in the parietal areas on both sides of the brain.

The development of these brain functions that are domain-specific, depends on the maturation of numerous functions including attention, working memory, language, sensorimotor function (or for example when using finger counting) and visuo-spatial ideation.

Their development depends also on the experiences, for example from everyday life and the type of teaching method used.

The cognitive functions that are related to number processing that, when affected, can cause the mathematical difficulties are:

- the working memory: that allows the maintenance of intermediate results of numerical procedures or of action plans needed to resolve complex calculation (Geary, 2012).

A reduction of the ability to process serial order in working memory has as result an abnormal processing in number tasks and can cause the absence of SNARC effect (Spatial Numerical Association response Code) that reflect the faster left than right hand responses to small numbers and faster right than left hand responses for large numbers (Dehaene et al.,1993; Ansari et al., 2008) that leads to low math score.

- the executive functions: that allows the monitoring of task performance and the control of the processes to improve future performances to avoid error and self-correction (Botvinic et al., 2007). Another executive function is the inhibition of inappropriate solution and mental processes to avoid intrusive errors and interference in simple math activities.

In sum mathematical problem solving is built on multiple cognitive components that are implemented by distinct and overlapping brain systems, impairment in any of these components compromises the efficiency of numerical skills.

Such a multicomponent system can explain the heterogeneity and comorbilidities observed in DD.

fMRI (*Functional Magnetic Resonance Imaging*) studies show that multicomponential neuronal networks subserve number processing and arithmetic and each subserving specific cognitive processes are together necessary to an optimal number processing.

The neural systems that are involved in number processing are:

- the visual, auditory association cortex, which helps decoding the visual form and phonological features of the stimulus, and the parietal attention system which help to build a semantic representation of quantity (Dehaene, 2003);
- the procedural and working memory systems anchored in the basal ganglia and fronto-parietal circuits (the intra parietal sulcus, the supermarginal gyrus in the parietal cortex, the premotor cortex, the supplementary motor area, the dorso lateral pre-frontal cortex) that, creating a short-term representation, supports the manipulation of discrete quantity and assure as said before, the optimization of the performances by monitoring them and inhibiting the incorrect responses;
- the episodic and semantic memory system (medial and temporal cortex and anterior temporal cortex and angular gyrus within the parietal cortex) that has an important role in long term memory and in generalization starting from individual problem characteristics;
- prefrontal control processes that are necessary to maintain attention to subserve the goal-directed decision making.

Neuroimaging researches that are focused on the investigation of neural of DD is still limited, but the most common findings is that at the brain level DD is characterized by atypical structural properties of the right intraparietal sulcus, and atypical functional modulation of this region during basic numerical processing.

DD children show an increased activation in the bilateral intra parietal sulcus, a weaker and more diffuse activation in the left intra parietal sulcus and this may suggest that the behavioral deficits in DD may depend on an impairment in the numerical magnitude representation through Arabic digits.

These findings and those given by neuropsychological and cognitive models are crucial to understand the nature and origin of DD for developing efficient remediation programs and didactic choices.

Typical development of numerical and arithmetical skills

The numerical intelligence is the ability to "intelligere" the quantity and is due both to innate abilities (Gellman and Gallister, 1978) and both to the strengthening of proximal development through education of domain specific processes (Fuson et al., 1983). The human species, even before they know how to count, can understand the phenomena in terms of quantity.

The innate numerical intelligence, does not exist only in humans, but in other species and the children of a few months (three months) are already able to perceive the numerosity of a set of visual objects, without knowing how to count (quantity distinction: one is different from many) and it is at the basis of various phenomena of different complexity (eg. plural, singular). Number sense (Approximed Number System) is innate, universal, language-independent, modal and abstract because the numerosity is mentally represented in an analogical format.

Numerical intelligence includes: the understanding of numerosity, the concept of number and the semantic encoding of the number (mentally represent the quantity that the number represents) and then to identify the position it assumes within the line of numbers. This numerical quantity system is activated when either the task is presented in analogical format and when it is presented through its representative symbols.

Among the innate abilities that contribute to the formation of numerical intelligence, there are subitizing and estimate. Subitizing is a non-intentional analogical act, a visual feature that allows a fast and accurate numerical judgment executed on numerosity of small sets of elements, what is defined as the ability "to count at a glance", but only for a maximum quantity of six elements. The estimate is a numerical process and is the ability to determine roughly and not counting unknown values (great quantity) the greater is the quantity more the approximation is used.

Other innate abilities are: the one to one correspondence, the counting of n. + 1 and n. –1 and the semantic access through the preverbal recognition mechanisms of the quantities that process the learning of reading and writing numbers and counting systems and give rise to the calculation mechanisms and to the manipulation of the number system. This is a conceptual representation that corresponds to the "meaning" of a number.

The numerical knowledge is due to semantic mechanisms (recognizing and manipulating quantity), syntactic mechanisms (the same range in several orders of magnitude) and lexical mechanisms (say, read and write numbers).

Counting is essential and a first link between the innate ability of the child to perceive numbers and the most advanced mathematical achievements of the culture in which he was born. Learning the sequence of the words used to count is the first way in which children connect their innate concept of number with the cultural practices used in their society. The implicit principles of counting are: the numerical lexicon and counting.

According to Gellman and Gallistel (1978) it is possible to highlight these different numerical abilities in the developmental progression:

- the one to one correspondence that is the ability to associate words-number to objects and separate the objects that have just been counted from those that have to be counted;
- the ability to understand the stable order, that is, the ability to use the numerical, verbal sequence in a stable and correct way;
- the recognition of the cardinality that is the knowledge that the number of the elements of a set is the last number used to count them:
- the ability of abstraction that allows to understand that any set composed of discrete elements can be counted;
- the ability to understand the irrelevance of order that allows to understand that the order in which the items in the collection are counted does not affect the result.

The syntactic mechanisms regulate the positional value of the digits and constitute the internal grammar of the number that activates the correct order of magnitude of each digit. In verbal encoding of a number each digit takes a different "name" depending on the position it occupies. In systems of understanding and/or production of the numbers, the lexical mechanisms have the task of adequately select the digit names to recognize that of the whole number.

To summarize, we could say that understanding, comparison, seriations, subitizing and estimate are the semantic aspects of numerosity, while production, reading and writing numbers represent their lexical and syntactic aspects. The cognitive processes of the computing system are therefore: semantic processes, lexical and syntactic and are represented by the ability to "counting", to calculate in mind, to retrieve the number facts and to produce a written calculation.

Counting is the ability to count, it foregoes the ability of calculation, is based on innate knowledge (n+1, n –1) and provides the ability to number forward or backward with reference to the quantity and to number for two or more.

Mental calculation is the ability to perform calculation mentally, it starts from the calculations (explicit count on the fingers), passes on to counting (counting from a given number) and is in need of effective learning strategies (breakdowns, rounding to the nearest ten, etc...).

The numerical facts relate to the ability to use simple operations already resolved encoded in memory, ready to be retrieved automatically if necessary (2 + 2, 3 + 2, 5 + 5, 50 + 50, ...), they include the multiplication tables in their entirety and not only the results and are the basis for facilitating the acquisition of facts such as the commutative property and, based on automatisms, they speed up the calculation processes.

The calculation script requires numerical knowledge (vocabulary, syntax, and semantics), calculation skills, the ability to perform strategic or automated facts and specific procedural knowledge (algorithms of the 4 operations). These evolutionary processes are sequential and must be learned following their evolutionary order.

Possible didactic choices

What are the useful proposals for teaching arithmetic in a learning early stage?

During the kindergarten the activities planned by the teachers should aim to develop cognitive skills of innate and analogical basis, so without the use of written symbols. Teachers should provide and organize activities that stimulate the upgrading of skill that forego those of computing and digital manipulation.

These activities should develop and stabilize:

- the subitizing and numerical estimate (an example is the use of games that require the use of dice or card games like the Italian game “Scopa”);
- the ability to one-to-one match, for example, to match the quantity and name of the number (enumeration) thanks to which the child learns to accompany the word number at the time of the count or to make the correspondence between sets in relation to the number of the elements;
- the knowledge of the size (the concept of cardinality, the ability to rearrange sequences compared to a characteristic or to the size of the elements);
- the first forms of calculation without using numerical symbols, but using an iconic representation of the quantity.

During the kindergarten the numeric symbols should not be used and the association with the quantity and the name of the numbers should be presented only during the primary school.

Other important activities to assure the numerical cognition development, at this institutional school level should reinforce the ability to control the coordination of eye and hand movement, assure the control of the graphic motor skills, the visual perception, the adaptation of the visual motor system to the graphic gesture for writing numbers and the development of visual and auditory memory needed for encoding and decoding written and listened symbols.

It should be also necessary to develop the knowledge of topological concepts like: before/after, long/short, big/small, up/down, right/left that are the prerequisite skills for the understanding of place value of the digit and the written calculations.

Other logical skills that should be acquired and enhanced at this school level are those of classification, of rhythm reproduction, the ability to create associations, logical relationships, the ability to reflect on the various possibilities and solutions with respect to problematic situations (problem solving).

Many recent scientific studies have shown that the movement has a cognitive value, for example, the study of Fisher et al., (2015) proved that the movements that involve the whole body, such as moving on a line of numbers drawn on the floor, can improve the efficiency of numerical trainings by improving the level of performance in multi-digit additions. This is another important indication that along with the other ones, should be the basis of teaching methodological choices at kindergarten school level, to ensure a development of the analogical and pre-verbal cognitive abilities of the numerical cognition. An inconsistent development or lacking of these skills does not grant the acquisition of the basic skills and becomes the basis of the difficulties in arithmetic in subsequent grade levels.

The child who comes to the Primary School must meet and master the concept of number, he must have acquired the numerical lexicon, must know how to count, use and apply simple calculation strategies.

If these skills are unreached,before proceeding with the subsequent learning,it is necessary to ensure to the child to perform activities to recover the prerequisites until they are fully achieved.

During the first year of Primary School before learning how to quickly and accurately calculate the child needs to develop and achieve a complete mastery of counting skills, of semantic, lexical and syntactic processing of the number through the use of numerical symbols, in encoding and decoding activities.

In the acquisition of the ability of mental counting, the choice of methodology and educational activities targeted to the acquisition of automatic calculation are very important.

An analogous method, based on visual recognition of submultiples of ten (the properly developed subitizing skills recognize at a glance five elements) is in line with the real brain functioning and facilitating mental math.

So, all activities involving the calculation at a glance of the quantity (sets, abacus, “Regoli”) are believed to be facilitating the computing activities.

A very recent study (Nuñez and Fias, 2015) has revealed that there is a line of mental numbers in the human brain, which has an ancestral match in the brain of other species (chicken for example) and then use the number line as one of the teaching methods for counting acquisition can only be a functional choice.

Also at this school level, physical activities will continue to be a consistent choice in strengthening the acquisition of numerical cognition and should not be considered as mere physical activities, but they have to be considered as real cognitive activities.

Activities to improve working memory appear to be equally functional to improve the arithmetic performance of the calculation and should be considered as an integral part of the mathematical learning program (Bergman-Nutley and Klingberg, 2014).

When teaching the graphics trace of the numerical symbol the teacher should make sure that children reach the ability to distinguish the signs without error, through multisensory and motor activities, to avoid the effects of symmetry and make it easier to distinguish the symbols that are visually similar (2/5; 6/9).

Recognize blindfolded the number made of cardboard just by touching it with the hands, use the whole body to represent the digit, write it using different materials (cotton, wool, buttons...), use the clay to write it and other similar activities will create strong visual images of the numerical symbols and allow a proper decoding and encoding of them, avoiding any confusion in their recognition and production.

Numerous scientific researchers have directly related the use of fingers to the areas of the number (Andrea et al., 2008; Fisher, 2008; Crollen et al., 2011) and therefore, the use of the fingers for counting and calculating should be promoted and maintained at least until the automatic calculation mechanisms are not established, as they enhance the development of arithmetic skills and are not the expression of its immaturity.

The number zero should be presented as a number that represents the lack of quantity and not like a vacuum because an educational choice like this conveys and facilitates the awareness of place value of Arabic numbers by assigning to the zero a place in the space and a value related to it.

Direct learning principle, which includes the demonstration, explicitly by driving, modelling, verbalization and reinforcement should be one of the fundamental bases of the school method since the findings indicate that the obtained results in terms of learning, making this methodological choice, are an improvement in accuracy and response times in every child, with or without computational difficulties (Goldman, et al., 1988).

All this information is at the basis of the methodology and didactic activities proposed by the Method "Libera…mente imparo" for the kindergarten and the first class of Italian Primary school used in numerous Italian schools.

The Method "Libera...mente imparo" for teaching reading, writing and calculating

In these last twenty years Scientific research has been interested in cognition and has received an unprecedented boost propulsion, due to the use of neuroimaging techniques in vivo, which have allowed us to understand the underlying brain.

Cognitive Science is the field of research where all the sciences that are interested in cognition came together for a comprehensive and practical approach to understand the mechanisms of learning and behavior that characterize the human being and its specific functions. In Cognitive Science converge the hard sciences of cognition, such as Neuroscience and Computational Science that attempt to reproduce the human brain mode, through the use of computer language, but also the human sciences such as psychology, philosophy, language and even the ethology. At the end of the 90s, some scholars as Bruer (1997), Stanovich (1998), Caine (1990) and others, have begun to question whether and in what way, the knowledge of how the brain learns could be useful for educational Sciences. The main issue was to decide how to bridge the gap between the information provided by the hard sciences, which are interested in the individual neuronal spike and Educational Sciences which are interested in the human being in its complexity.

The debate was very heated and still continues today. The different scholars (Caine, 1998; Ansari, 2005, 2006; Goswami, 2015) have given different answers, but all have emphasized the enormous usefulness that Neuroscience may have for Educational sciences since the information they provide are considered useful for important purposes such as finding solutions to problems related to the evolutionary development of children; understanding long-life, introducing educational changes in the practical application of scientific discoveries that come from neuro-imaging techniques.

According to the neurolinguist Jodie Tommerdhal of the University of Texas (2010) to bridge this gap four phases are necessary. The first step is the level of Neuroscience that studies the brain at the level of its cells and how information is transmitted, during the perceptual-cognitive activities. The next level is that of Cognitive Neuroscience that studies the functions and brain architectures that can be connected directly to cognition, using the neuroimaging techniques. The level below is that of the biological mechanisms that focus on the connection between physical actions and functions, the psychological mechanisms underlying cognition.

The level of educational theories is that in which the teaching and learning theories are developed based on the information from the previous levels.

Once the method is structured in theory, it goes to the last step of the way, that is, the level of the class in which the new teaching methodological proposals are tested through research into action.

On the basis of these indications a research project called “Cervello, Cognizione & Educazione” was created in Italy whose purpose is to create a link between the information coming from Cognitive Sciences and Educational Sciences to create teaching methods based on the actual brain functioning, that can ensure an inclusive approach in teaching practice.

The idea of inclusion that comes from this perspective is very innovative, because it is based on the multiplicity of information coming from Cognitive Sciences that, studying the neurobiological basis of learning, may provide continuously updated information on brain mode “default”, common to any type of learning, such as reading or speaking a second language or arithmetic. The final products of this research, the teaching methods, are inclusive just because based on the basic brain functioning that guarantees the acquisition of the fundamental biological knowledge necessary to learn and moreover, it allows any individual approaches to compensate both specific learning difficulties and cultural or linguistic difficulties those that are defined Special Educational Needs in the school context.

The research path, born in 2010, as the Doctoral Research project and final thesis of the author in Cognitive Sciences at the University of Messina, is the only one, for now, in Italy and within the international panorama, that has brought to completion all the phases needed to create a method of teaching reading, writing and calculating starting from the information given by the Cognitive Sciences through Educational Sciences by means of the experimentation in the class by research into action.

The first product of this research is the method of teaching reading, writing and calculating for the first class of Italian Primary school called “Libera...mente imparo” (Collerone, 2011, 2012, 2013, 2014), implemented in research into action in the years school 2008/2010 and which has now been adopted in numerous schools in Italy, by teachers open to innovation and new educational pathways.

The results in terms of learning were evaluated at the end of the second class of the Primary school with the national “INVALSI” test and have demonstrated the effectiveness of the Method in achieving the learning objectives set by the Education Ministry. These results exceed not only those achieved by children in parallel classes, where different teaching methods were used, but they also outnumber the results at regional and national level for the corresponding classes.

The Method has gained recognition in academic circles both at the national and international level and is currently in use in several Italian schools.

The information from the Cognitive Sciences are constantly changing, the teaching activities that operate on the brain changes must always take into account that the brain is plastic, and adaptableand is influenced by the environment and for these characteristics, constantly evolving. Moreover, teaching must on a very complex system (social and environmental), where there are many variables and each variable involves a change to the entire system.

Teachers should accept the idea that their methodological choices and their teaching actions must take into account the change, the evolution, the concept of development and "in progress" and they have to abandon a rigid and static approach to teaching and learning.

They have to accept that they need to constantly update what they know, their choices and the means used in their teaching activities as to allow each student, the first variable in the learning process, to learn in connection with the peculiarities of his individuality, in the respect of the fundamental constraints that the brain structure imposes.

Conclusion

This article outlines the view that number processing and mathematical problem solving is built on multiple neurocognitive components that are implemented by distinct and overlapping brain systems. Impairments in any of these components can compromise the efficiency of numerical problem solving skills.

The teaching method has a strong impact in early acquisition of numerical cognition, and if it takes into account the information given to typical and atypical brain functioning in learning arithmetic to build the methodology and choose the didactic activities, it can grant an appropriate learning of the skills at the basis of an adequate numerical cognition.

Future research should continue to gather information on brain functioning and should have to turn them into methodological and educational choices that can be implemented in the context of the class, to test their effectiveness in real conditions of teaching activities to ensure the development of new choices in line with the continuous proceeding knowledge in the field of cognition, functional to the creation of an inclusive education because "brain based". The teaching methodology cannot erase the atypical brain functioning that are characteristics of Developmental Dyscalculia, but it can greatly contribute to contain and compensate its effects on learning.

Declaration of Conflicting Interests

The authors declared that they had no conflicts of interest with respect to their authorship or the publication of this article.

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References

Andreas, M., Di Luca, S., Pesenti, M. (2008) Finger counting, the missing tool? *Behavioural and Brain Sciences*; 31, pp. 642-643.

Ansari, D. & Coch, D. (2006). Bridges over troubled waters: education and cognitive neuroscience, *Trends in Cognitive Sciences*; 10, 146 151.

Ansari, D. (2005). Time to use neuroscience findings in teacher training, *Nature*; 437, 26.

Ansari, D. (2008). Effectsof development and enculturation on number representation in the brain. Na-ture*Reviews Neuroscience*; 9:278–91.

Ashkenazi, S., Rubinsten, O., Henik, A. (2009a). Attention, automaticity, and development dyscalculia. *Neuropsychology*; 23: 535–540.

Ashkenazi, S., Mark-Zigdon, N., Henik, A. (2009). Numerical distance effect in developmental dyscalculia. *Cogn Dev*; 24: 387–400.

Bergman-Nutley, S., Klingberg, T. (2014) Effect of working memory training on worling memory, arithmetic and following instructions.*Psychological research*; 78:869-877.

Botvinick , M., Watanabe, T. (2007) From numerosity to ordinal rank: again-field model of serial order representation in cortical working memory. *The Journal of Neuroscience*; 27: 8636–42.

Bruer, J.T. (1997). Education and the brain: A bridge too far. *Educational Researcher*, Vol. 26, No. 8, pp. 4-16.

Butterworth, B. (1999). *The Mathematical Brain*. Macmillan, London.

Caine, R.N., & Caine, G. (1990). Understanding a brain-based approach to learning and teaching. *Educational Leadership*;48(2), 66-70.

Caine, R.N., & Caine, H. (1998). Building a bridge between the neurosciences and education: Cautions and possibilities. *NASSP Bulletin*; 82(598), 1-8.

Collerone L.M. (2011). Il cervello che legge "Libera…mente impara". *Educare. 7*. Doi 10.4440/20111106/collerone.

Collerone, L.M. (2012). L'educazione Neuroscientifica: collegare le scienze cognitive all'educazione. *Atti del nono convegno AISC*, pag. 27-33. Curato da F.Cecconi, M. Cruciani, Editore UniTrento.

Collerone, L.M. (2013). Il riuso neuronale come fondamentale principio riorganizzativo del cervello:dalla teoria biolinguistica alla pratica sperimentale nel campo della Educazione Neuroscientifica. *Atti del 6° Convegno Nazionale dei Dottorandi di ricerca CODISCO 2012 "Animals, Humans, Machines. The Whereabouts of Language"*24-26/09/2012 Uniroma3.

Collerone, L.M., Città, G., Buttafuoco, G. (2014). Progetto di ricerca "Cervello, Cognizione & Educazione" un approccio multidisciplinare per la creazione di metodologie didattiche "brain friendly" per l'apprendimento della lingua inglese scritta per bambini della classe prima della scuola primaria. *Rivista Mondo Digitale di Didamatica* pp.466-471 ISBN 978-88-98091-31-7.

Crollen, V., Serou, X., Noël, M.P. (2011). Is finger-counting necessary for the development of arithmetic abilities? *Frontiers in psychology* (published on line).

Dehaene S., Piazza, M., Pinel, P., Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology*; 20: 487–506.

Dehaene,S., Bossini, S., Giraux, P. (1993).The mental representation of parity and number magnitude. Journal of ExperimentalPsychology:General1993;122: pp. 371–96.

Fisher, M.H., Brugger, P. (2012). When digits helps digits: spatial-numerical associations point to finger counting as prime exampleof embodied cognition. *Frontiers in Psychology*; 2,389, pp.41-47.

Fisher, U., Moeller, K., Huber, S., Cress, U., Nuerk, H. (2015). Full-body movement in numerical training: a pilot study with an interactive whiteboard, *International Journal of serious games*; Vol.2, N. 4.

Fuson, K.C., Secada W.G., and Hall J.W. (1983). Matching, counting, and conservation of numerical equivalence. *Child Development*; 91-97.

Geary, D.C., Hoard, M.K, Bailey, D.H. (2012). Fact retrieval deficits in low achieving children and children with mathematical learning disability. *Journal of Learning Disabilities*; 45: pp.291–307.

Geary, D.C. (2004). Mathematics and learning disabilities. J Learn Disabil; 37: pp.4–15.

Gelman, R., Gallistel, C. (1978). *Young children's understanding of numbers*. Cambridge, M. A. Harvard, University Press.

Gozuyesil, E, and Dikici, A. (2014). The Effect of Brain Based Learning on Academic Achievement: A Meta-Analytical Study, *Educational Sciences: Theory and Practice*; 14, 2, pp.642-648.

Jordan, N.C., Montani, T.O. (1997). Cognitive arithmetic and problem solving: a comparison of children with specific and general mathematics difficulties. *J Learn Disabil*; 30 (624–634): 684.

Jordan, N.C., Hanich, L.B. (2003). Characteristics of children with moderate mathematics deficiencies: a longitudinal perspective. *Learning Disabilities: Research and Practice*; 18: 213–221.

Kaufmann, L., von Aster, M. (2012). The diagnosis and management of dyscalculia. *DtschArzteblInt*; 109(45): pp.767–78.

Krause, B., Márquez-Ruiz, J. and Cohen_Kadosh, R. (2013). The effect of transcranial direct current stimulation: a role for cortical excitation/inhibition balance? *Frontiers in Human Neuroscience*; 7:602.

Kroesbergen, E., van Luit, J.E.H. (2003). Mathematics Intervention for Children with Special Educational Needs. *Remedial and Special Education*; 24: 97–114.

Kucian, K., von Aster, M.G., Loenneker, T., Dietrich, T., Martin, E. (2008). Development of neural networks for exact and approximate calculation: a fMRI study. *Dev Neuropsychol*; 33(4): 447–73.

Lenhard, A., Lenhard, W., Klauer, K.J. (2011). *DenkspielemitElfe und Mathis.Förderung des logischenDenkvermögensfür das Vor- und Grundschulalter*. Göttingen: Hogrefe.

Marshall, E., Mann, V., & Wilson, D. (2006). *Maths anxiety*. http://www.sheffield.ac.uk/mash/anxiety

Mussolin, C., Mejias, S., Noel, M.P. (2010). Symbolic and nonsymbolic number comparison in children with and withoutdyscalculia. *Cognition*; 115: 10–25.

Nuñez, R., Fias, W. (2015). Ancestral Mental Number Lines: What Is the Evidence? *Cognitive Science*; 1-5.

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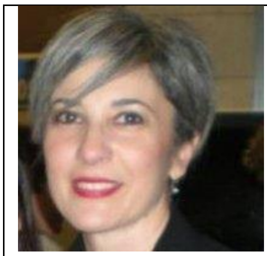

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Flipped learning – Why teachers flip and what are their worries?

Marika Toivola

Abstract. *Flipped learning (FL) in mathematics has recently attracted considerable interest around the world. It raises the question of whether humanist perspectives, such as freedom, dignity, and potential of humans, should have a higher profile in teaching and students' learning in mathematics. This article has used data collected from an open questionnaire where 33 Finnish teachers illustrate why they flip their learning culture in the first place and what are they worries about flipping. The main aim of this article is to offer theoretical and emotional support for teachers' enthusiasm to change their learning culture in a more student centred approach and raises the discussion about what flipped learning really is. What are its pedagogical assumptions? After strengthening our theoretical understanding about why flipped learning seems to be functioning so well we can also better understand what is central in the approach to what we are trying to achieve by applying it.*

Key words. *Collaborative learning, differentiation, Finland, Flipped classroom, Flipped learning, self-regulation.*

Abstract. *Il Flipped learning (FL) in matematica ha recentemente riscosso grande interesse a livello mondiale.*

Ci si pone l'interrogativo se valori quali la libertà, la dignità e il potenziale umano debbano avere un maggior rilievo nell'insegnamento e nell'apprendimento della matematica.

Il presente articolo si basa su un questionario aperto nel quale 33 insegnanti finlandesi illustrano il motivo per cui capovolgono il loro habitus di insegnamento in primo luogo e quali sono i loro dubbi a tal proposito.

Lo scopo principale di questo articolo è quello di fornire sostegno teorico ed emotivo agli insegnanti al fine di favorire un cambiamento nel modo di apprendimento che divenga più centrato sul discente e al contempo fa emergere la discussione su cosa si intenda realmente per Flipped Learning e su quali siano i suoi presupposti pedagogici.

Key words. *Collaborative learning, differentiation, Finland, Flipped classroom, Flipped learning, self-regulation.*

Introduction

These days, Flipped Learning (FL) fascinates many teachers who are trying to change their learning culture to a more student-centered learning approach. The ideology of flipping has started to develop from a method of teaching known as flipped classroom (FC). FC is seen as a concrete step to change the dynamic of class and step out of the more traditional learning culture, in which class time is typically spent on direct instructional guidance, while the most higher-order tasks are done as homework, outside the classroom. However, FC does not guarantee a learner-centered learning culture because it simply puts the focus on moving tasks in space and time, rather than on increasing engagement, autonomy or student centeredness (Abeysekera & Dawson, 2015). In this article, FL is neither a synonym for FC nor the same as FC from students' point of view. Instead, FL is understood as flipping not only the actions in a classroom, but also many of the teachers' pedagogical assumptions about teaching and learning (Toivola & Silfverberg, 2015). In conclusion, in this article FL is understood as an ideology of learning, not a method of teaching.

Since 2007, Chemistry teachers Bergmann and Sams are widely known as pioneers of flipped classroom (FC), when their students started to watch recorded lectures at home and do exercises in class under supervision. Their story presented here illustrates the nature of flipping as a continuously developing learning culture. In the first book of Bergmann and Sams "Flip Your Classroom: Reach Every Student in Every Class Every Day" published 2012, their ideology of flipping got a name Flipped-Mastery model as a purpose to differentiate it from the more traditional FC which was typically connected to learning videos. The name of their second book "Flipped learning: Gateway to Student Engagement" (2014) illustrates a further development of flipping ideology and how flipped classroom as a teaching method is not enough for reverse to meet the student centered learning culture. Here are some quotations where Begmann and Sams (2012) describe how the process of flipping has transformed not only actions in their classrooms but also their thinking about education: "The best analogy [for teacher] we can come up with is the role of a supportive coach. We are there to encourage our students along the road of learning. They need a coach who can come alongside them and guide them in the discovery of knowledge. … Learning is no longer imposition on their freedom but rather a challenge to be unpacked and explored. As the teacher gives up the control of the learning process, the students take the reins, and the educational process becomes their own. ... Our classes have become laboratories of learning where the entire focus of the classroom is on what students have or have not learned. No longer do we present material, provide a few extra learning opportunities, to give a test, and hope for the best. Instead, students come to class with the express purpose of learning. We provide them with all the tools and materials to learn, and we support them by helping them to develop a plan for how and when they will learn. The rest is up to the student. ... When learning becomes the central point of the classroom, the students must work just hard as the teacher. This means their minds are engaged, not just passively exposed to information."

In 2014, the Flipped Learning Network, which is a professional learning community for educators using FL (has over 28 000 members on its social media site www.flippedclassroom.org), and Pearson's School Achievement Services (Yarbro, Arfstrom, McKnight, & McKnight, 2014) defined FL as a "pedagogical approach in which direct instruction moves from the group learning space to the individual learning space, and the resulting group space is transformed into a dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter". They also identify four components that support students' engagement, which are generally known as the four pillars of F-L-I-P (Flexible Environment, Learning Culture, Intentional Content, and Professional Educators). Later in the same year Chen et al. (2014) offered three additional letters, P-E-D (Progressive Activities, Engaging Experiences, and Diversified Platforms), to the F-L-I-P schema to emphasize the activities-oriented nature of FL. They criticized F-L-I-P of focusing 'existing knowledge assimilation' rather than 'new knowledge discovery and creation'.

Like the name Flipped-Mastery model illustrates FL has raised the basic tenants of *mastery learning* (Bloom, 1968; Carrol, 1963) by re-considering and evaluating processes. Already almost a half century ago Bloom argued that the level of learning determined based on teacher conceptions of the average skill level of the group will be poorly fit to 80% of the students. Mastery Learning is very logical, fascinating and is one of the most highly investigated teaching methods over these years. However, it has also been the subject of high criticism and many mastery programs have been reconsidering the more traditional way of teaching due to the level of commitment required by the teacher and the difficulty in managing the classroom of individual learners (Guskey & Pigott, 1988). In FL, *Self-paced learning* and *scaffolding* are placed at the center of the learning culture as well as ICT is harness to support learning. The time will show if FL succeeds in taking up the gauntlet of mastery learning.

Even though the FL movement was created from teachers' common sense thinking and enthusiasm to just jump into the new learning culture, rather than from educational theory found in literature, in their study Toivola and Silfverberg (2015) have shown that the target of FL is closely linked to classical research questions about education. However, these questions are not easy for teachers to comprehend because they are questioning the role of the teacher. The main aim of this article is to offer theoretical as well as emotional support to teachers in their developing process of FL. As data for this study an open questionnaire has been used, in which 33 teachers have answered. When a teacher flips the learning culture he or she is not questioning only the learning culture of his or her own but inevitably also the learning culture of other teachers in the same school. For this reason, teachers are worried about the depression caused by the resistance of colleagues as well as the resistance of students and their parents.

The Finnish context

In Finland, where this research was made, there are more than 10 000 members in a Facebook group where teachers can collaborate and share ideas about flipping. In 2015, the first two-day long summer meeting for flippers was arranged to offer the teachers who were familiar from social media a place for face-to-face collaboration. In the meeting the teachers were enthusiastic to keep short presentations about their learning culture and share their successful as well as unsuccessful practical experiences. There were 60 attendees and 33 from those answered the questionnaire, which have used as a data in this article. In addition to some background information the questionnaire has only three open questions: What flipped learning means to you? Why you flip? What are your worries in the changing of the learning culture? As a notation, these teachers do not represent the average teachers in Finland. Instead, they are highly motivated to develop their learning culture and joined the course by their own will in the second week of their summer holiday. The summer meeting was held in Mikkeli, which is about three hour drive from Helsinki. The attendees were primary teachers, special class teachers, secondary and upper secondary level teachers, and principals. The following school subjects were represent: mathematics, physics, chemistry, information technology, literature, English, French, religious education, biology, geography and guidance counseling.

The Finnish way of flipping is quite similar with the method of Bergmann and Sams's Flipped-Mastery Classroom which follows the principles of mastery learning connected with modern technology to make a sustainable, reproducible, and manageable environment for learning. Thus, in Finland because of teachers' autonomy and lack of standardized tests teachers have freedom to concentrate even more to the learner-centered approach (Tangney, 2014), which consists of both constructivist and humanist elements, such as personal growth, increased consciousness, and empowerment.

Teachers' reasons to flip and their worries about flipping

Based on the data the most important reasons for teachers to flip were

(1) differentiation,

(2) motivation and self-regulation, and

(3) collaborative learning.

In the next chapters these reasons and worries connected to them will be opened and theoretical justifications will be discussed. In the final chapters flipped assessment will be introduced and focused on teachers' emotional support.

Dirretentiation and the quality to the students' zone of proximal development

Differentiation is a buzzword in today's educational community. Twenty teachers out of 33 said differentiation to be the most important reason to flip. "*Both the weak and gifted are benefiting in FL. The students have possibility to practice those skills, in particular, which they need at that moment.*" Even though self-paced learning delights teachers it also distresses. "*If I let students self-pace their learning, what happens if the next teacher won't? Do the students get over the minimum requirements of the curriculum? How much will be enough? Do we have time to go through all?*"

Because the teacher could neither be able to affect the heterogeneity in the class nor to choose the level of learning suitable for all the students, the teacher just stops to act like he or she could. In FL the teacher lets the students differentiate their own styles and goals of learning by themselves. The students simply have the possibility to start to learn from a level which is best fitting their own *zone of proximal development* (ZPD) at the moment.

The teacher role is to impact the quality of the students' ZPD.

In his theory, Valsiner expands the ZPD to include two additional zones of interaction: the *zone of free movement* (ZFM) and the *zone of promoted action* (ZPA) (see Fig. 1). ZFM is a socio-culturally determined function of what the students are allowed by the teacher and not prohibited by the learning culture they have engaged. Whereas ZPA defines a set of actions in the environment by which the teacher attempts to persuade students to act in a certain way. In figure (see Fig.1), Toivola and Sifverberg (2015) illustrate the quality of students' ZPD in FL by using the *illusionary zone of promoted action* (IZ) first launched by Blanton's et al. (2005). As opposite the Blanton et al., who use IZ as a diagnostic tool for visualizing the teacher's own ZPD, Toivola and Silfverberg use IZ as a zone which students can achieve without the teacher's promotion if students' spontaneous performance is allowed. However, this point of view does not exclude IZ as a tool for visualizing the teachers' own ZPD too. Bergmann and Sams (2012) comment, "Our classes have become laboratories of learning", is excellent manifestation of IZ especially from the teacher's perspective.

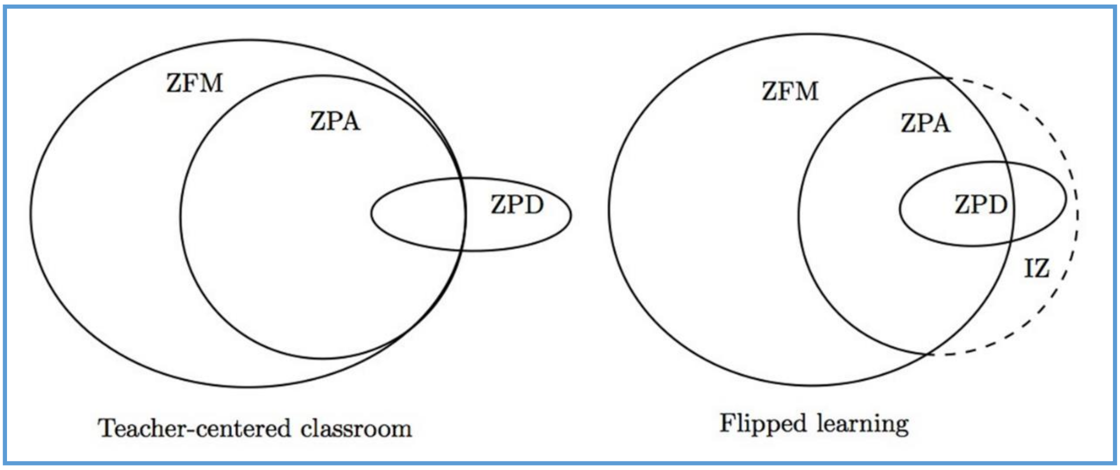

Fig. 1 - The zone of proximal development (ZPD), the zone of free movement (ZFM) and the zone of promoted action (ZPA) in a traditional teaching context (adapted from Oerter (1992)) and in a flipped learning context (the authors' interpretation by exploiting Blanton's et al. (2005) illusionary zone of promoted action (IZ)). (Toivola & Silfverberg, 2015)

Student's ZPD is not fully contained in ZFM/ZPA complex as a result of teacher's decisions about what he or she allows the student to do in situations of a different kind. For example, if a teacher does not allow students to participate during exercises, he or she undoubtedly excludes the students' possibility for reciprocal scaffolding (will be explained in the next chapter). This does not mean that teacher should never use individual seatwork exercises. Instead, the purpose is to illustrate how some distinctive teachers' instructional choices will reflect the students' learning environment. There is no doubt that sometimes for example direct instruction has a place also in FL, however, the place neither needs to be in the classroom nor in a whole-class setting. In FL teacher promotes the quality of students ZPD and the potential of students' development by the increased autonomy of the students and by focusing on them as individuals not a homogenous group. In other words, the teacher offers tools of different kind for learning (ZPA) and everything is not mentioned to everyone. What the teacher offers is an option and the student are free to choose differently as long as their choices serve learning. In FL, teachers were released from the duty that all should learn the same matters at the same time.

Motivation and self-regulation is connected to teachers control

24 teachers mentioned the purpose to flip is to increase motivation. According to them, the motivation rinses if students are allowed to take ownership of their own learning process and take responsibility of their learning. From students' point of view, flipping demands students to control their learning by taking responsibility for the pace of study, the mastery of the content, and coming to class prepared (Bergmann & Sams, 2012; Moore, Gillett, & Steele, 2014). However, eight of these teachers also felt they struggle with how to best help students achieve self-regulation in their learning. "*What if do students not self-regulate? How can I help passive and lacy students? What happens to those students who are unwilling to do anything by themselves?*"

Guiding students to become autonomous learners is neither an easy or a fast task. Demetry (2010) for example noticed that 10-15% of the students has lack of self-regulation in FL. That still suggests than an impressive 85-90% of the students did self-regulate. Nevertheless what is self-regulation and how to support it? Learning to self-regulate the learning process is connected with the teacher's conception of humanity and his or her perception of the student as a person (Koro, 1993). Finnish teachers try to respond to this demand by focusing on joy of learning and school satisfaction (15 answers of 33) as well as rising the students' self-esteem by offering more feelings about success and less comparison between classmates (5 answers). Purposely teachers who flip also concentrate on great learning environment where everyone feel good and where no one is left behind.

To support self-regulation students are allowed to take more control and responsibility of their own learning. In figure (see Fig. 2), Toivola and Silfverberg (2015) illustrate how self-regulation is connected to teachers' control. Students who are overly controlled by the teacher not only lose initiative, but also learn less. Self-regulation in learning and students' intrinsic motivation seem to increase when students have an opportunity to decide when they need personal guidance, encouraging feedback or non-authoritarian cooperation (Deci & Ryan, 1985; 2000). To be an autonomous learner, the student should be the leader, designer and implementer of the learning process, which means not only in *organizational* and *procedural choices* but also in *cognitive choices* (Stefanou, Perencevich, Di Cintio, & Turner, 2004). However, it is not easy for the teachers to give learning hands on students and stop prevent students from making pedagogically poor choices.

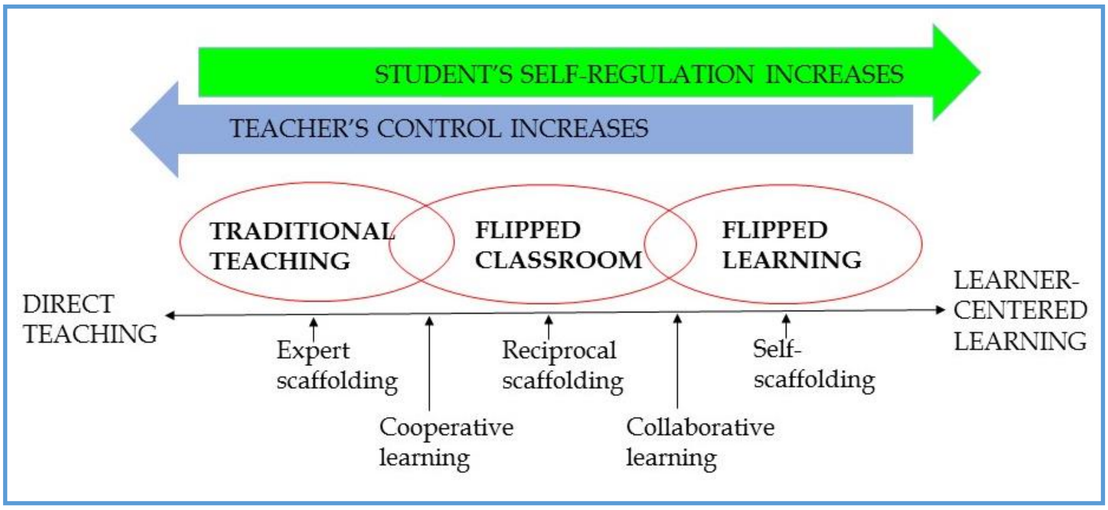

Fig. 2 - The dimensional view of the changes when switching from direct teaching to learner-centered learning. (Toivola & Silfverberg, 2015)

Quit often based on conversations in social media self-regulation is misunderstood as a synonym with self-study. The Self-regulated learning is not learning in isolation or alone. Instead, the significance of others in the development of self-regulation is explicit in the seminal works of both Piaget and Vygotsky. More recently McCaslin (2009) as well as Volet et al. (2009) highlighted the significance of the individual's metacognitive and scaffolded experiences in social systems and used the term *coregulation* to illustrate a transitional process in the development of self-regulation. Thus, to support self-regulation students should be motivated to collaborate each other.

Fifteen teachers mentioned they do not want to motivate only students but also challenge themselves. They see FL as a tool for modernizing their previous learning culture and grow as a teacher (IZ). With FL, they could see the potential of the students better and have possibility to affect their students' learning attitudes. However, as a total of 17 teacher form 33 were worried about their own role and how qualified they are to support student-centered learning. "*Can I direct learning? The change in the culture of learning is so big that it is challenging for me as a teacher? I have to learn to be active in the different way than in the traditional way of teaching. I really like to be the front of the class. I really like those conversations with all students where I am a leader. Will I be satisfied with a smaller role as a teacher*?"

Collaborative learning culture supports self-regulation

Fourteen teachers mentioned collaborative learning as a purpose to flip. Thus eight of them were also worried about how to promote collaboration. "*How do I promote collaboration at the level of the whole group? Do the individuality and the cooperation come true really? What can I do to help everyone to find a group? How noisy will the class be if there is so much social activity? What will I do with those students who have* *social problems and conduct disorders?*" It is clear that collaboration is an important factor in FL but what kind of collaboration? More precisely, what is the teachers' role in collaboration? What kind of collaboration supports self-regulation?

In their research Toivola and Silfverberg (2015) illustrate how the nature of the collaboration depends on teacher (see Fig. 2). With *collaborative learning* they referred to Dillenbourg's (1999) definition as a shared learning situation in which two or more people learn or attempt to learn something together. In contrast, *cooperative learning* focuses on a common or final output. In FL teachers give up control also in the matter of collaboration. Cooperative learning, where students are forced to work collaboratively to fulfil a task offered by the teacher, is perhaps not the best option if the purpose is students' self-regulation. To become self-regulative learners, the students should be the ones who are allowed to use collaboration the way how it best fits in their own needs at the moment to practice their *conceptual thinking* (Kazemi & Stipek, 2001) and to improve their *mathematical identity* (Anderson, 2007) without any obligation set by the teacher. A central but quite often neglected goal in mathematics education is to let students express their own mathematical thinking. Learning to use mathematics as a language is commonly based on imitating the teacher. As with learning any foreign language, only imitating based learning does not seem to be enough to achieve the active skills of using the language of mathematics. Research has shown that when students explain their problem-solving processes it develops their problem-solving strategies, and their metacognitive awareness of what they do and do not understand. Discussion leads to a deeper understanding and has a positive impact on the quality and quantity of learning (Webb et al., 2009).

Four teachers mentioned the ideology of learning to learn and metacognition as a reason to flip. Based on Holton and Clarke (2006) study, metacognition is equivalent to self-scaffolding. They divided scaffolding into three types: *expert scaffolding*, *reciprocal scaffolding*, and *self-scaffolding* which operate in two scaffolding domains: conceptual scaffolding and a heuristic one. Even though there are in FL also presence of expert scaffolding and reciprocal scaffolding, self-scaffolding can be seen as a purpose of collaboration to achieve metacognitive skills. Self-scaffolding is a form of internalized conversation where the students negotiate their epistemic self. Self-scaffolders are aware of, what they know in terms of content knowledge, heuristic knowledge and learning styles. This unique form of thinking, where a student turns the discourse-for-others to a discourse-for-oneself, needs collaboration to develop (Ben-zvi & Sfard, 2007). The conversations in the class are also important for the teacher by offering a window into the social process of students' self-regulation and further offering the hints to better serve students learning (ZPA).

Flipped assessment in student-centered learning culture

Five teachers mentioned worry about assessment. If the way we measure our students does not change, the learning culture will not change either. As sad it is students mainly seem to learn for assessments. It should be remembered that FL does not decrease the heterogeneity of the class. There are still those who have excellent abilities in math and those who do not. However, teachers do not need to close their eyes to heterogeneity either when they measure the students. Otherwise the nature of this article, I give an example of flipped assessment via my own students. Thus, I was a keynote speaker in the summer meeting and in that role suitable to introduce my own learning culture of secondary school mathematics. I have tried to take up the gauntlet to support student centeredness by flipping the assessment, too.

My students may chose three different level tests depending on the credit in students' sight: credit seven, credit eight or nine and credit ten (in Finland credits are from four (fail) to ten). The basic idea is that students decide by themselves which credit they court in math. My role is to help them to achieve their goals. Of course the credits must be earned, but the students are not doing a kind of "cooper test" where the game is blew over and the students are forced to continue to the next subject based on the schedule of the teacher. I do not want to support rote learning either.

There are no mandatory test days in my class. The students will make the test depending on the time of their studies. The students are advised not to read for the math test because if they do so just in the previous evening they will be "cheating" in the test. When the students are cramming for a test, they will store the information into their short-term memory and this way destroy their possibility to see what they really have learned and what they have in their long-term memory. As long as a teacher offers the students a test paper where they can just vomit their knowledge the students continue this culture of rote learning. However, to stop action which has previously lead good credits is not easy for students and sometimes hard for their parents to understand. Students really need to trust that the teacher will not penalize them because of their mistakes.

When the students have made the test, I will just mark with coloured pen the number of exercise where there are some problems. After that, the students are offered precious opportunities to relearn and remediate. The purpose of the test is to help students self-evaluate their learning, it is not a purpose for teachers to evaluate their students. Assessments are used solely to show students what they have mastered and to help them decide what to do next. The students decide how they will correct the wrong answers; with or without the text book and further alone or with the help of classmates or teacher. Because the students are doing the tests in different times I always have time to concentrate on every student as individual and give feedback.

In FL, the teacher gives up the illusion that everyone will learn everything. The flipped assessment continues the same ideology. Instead of feeding the illusion that everyone would have realistic chance to get the highest credit ten in the test, the teacher accepts the fact that not everyone has learnt everything. The purpose is not to bring the students down to earth and to situate them to Gauss curve based on their mathematical ability. Instead different level tests have been seen as steps to climb higher and higher the mountain of math. It depends on students' ability to understand math as well as their willingness to work how high they will go in their studies. Flipped assessment offers every students possibility to see themselves as true learners of mathematics regardless of their level. As a matter of mathematical identity, it differs if the students have had a possibility to earn credit seven or if they have just shown how far they are from credit ten. Of course I encourage the students to take more advanced test whenever it is appropriate.

We are using mathematics text books, which I have made with another mathematics teacher. The books are published with creative common licence (CC-BY) and made for flipped learning. Everyone can load, use and edit books for free (see more www.flippedlearning.fi). Teachers can edit and enrich the interactive learning materials by adding videos, learning paths, level tests and discussions for example. The students also have possibility to make the math books of their own and take those books with them to the next school level. The students differentiate their learning with three different level exercises: the ground level, intermediate, and advanced. Naturally, students who are practising also the most difficult exercises will take the credit ten test. The final credits to the school report will be decided in collaboration with students. The students will tell me which credit they have earned and why. My role as a teacher is to help students identify their skills with the help of flipped assessment.

Emotional support for teachers

Changing his or her own learning culture is not an easy task for a teacher. The teacher is not questioning only his or her own learning culture but inevitably also the learning culture of other teachers in the same school. In the summer meeting were teachers whose all colleagues in the same school were using FL in math. Thus, the most common was situation where a teacher felt of being a lonely wolf with this ideology of learning. It comes clear that behind every succeed experience there was support by the principal. This support is mandatory because there is a risk for the teacher to burn-out not only for the amount of work at the beginning but also because of depression caused by the opposition. As a total of ten teachers from 33 attendees mentioned they were worried about the resistance of their own colleagues. Two mentioned being worried about the resistance of the principal, four resistance of the parents and seven resistance of the students. Tim is 60-year-old upper secondary school mathematics teacher and I asked him to share his story with us, which is not rare:

"*I started in a new school in a new city almost three years ago. I knew nobody and nobody knew me. It was easy to "do your own thing". The ethos of most schools is that teacher closes the classroom door and is on her/his own. After few weeks of traditional teaching, I made decisions: No more assigned home work. No more using more than half of the lesson explaining homework to those who already mastered them and to those who didn't still get it. Then it was easy to cut frontal teaching to minimum. Soon I added a* *form to monitor the amount of problems students solved. At first it was purely for myself: a tool for seeing what was going on in the classroom. To my surprise, it turned out that walking around the class gave me enough information. Thereafter the form has evolved to a tool for students for monitoring themselves. Students are encouraged to evaluate each problem on a short scale: Did I just copy the answer, or did I learn something, or did I really nail it. I strongly try to make students learn to choose the level of their homework problems. They must be able to analyze and evaluate problems at some level. I can help to pick up problems from the text book, but it is their sole decision, what to do.*

I shared my opinions as well as my feelings on the classroom with anyone who listened in the teachers' lobby, but there was very little actual conversation. Teachers seemed to be interested in their families and their own hobbies. It was not a custom to share your opinions about learning and teaching. I was little miserable and felt myself an outsider. Then I found out there was a teacher of another subject (we are more than 40 teachers, a big teacher staff in Finland) using ICT and giving students a freedom of choice with their tasks at home and during lessons. That gave me the possibility to compare our ways of teaching.

There were still very few opportunities to reflect my decisions for arranging my teaching with other math teachers. There was a teacher I knew who was very active in the social media. I wrote some e-mails to him and he encouraged me to carry on. We also discussed the differences of our methods in more detailed way, a thing I couldn't do in at my own school. Later on I participated in a summer meeting where a group of teachers gathered to share their views on subjects like "how to make the student the subject instead of an object at school" and "how to enhance the using of ICT in a way that supports students learning in a new way instead of just digitalizing the old-school-teaching". That summer meeting really made my day.

Then there was a new fall and back to school...

I had a kind of grown to do my own ways of lesson planning and my own way of giving homework and my way of using videos and group work and so on. And I think that the other teachers politely let me do that, but nothing else changed in the school. Then out of the blue, the principal told me he appreciated a lot my input as a teacher and a developer, too. That gave me a lot of new energy to carry on and make a little better every time I had a new class in our 7 week periodic system.

I also started to get feedback from other teachers at our own school and from teachers from other schools via social media. Teachers recalled our discussions, we had long time ago. Teachers asked me to share my material with them (just to look through, but not to use at first). I realized the power of sharing materials and ideas in a Facebook group. Some feedback from the students showed they shared my views of learning math.

There seemed to be a profound change in my way of teaching, and also in the way I see teaching as a whole in these two years. It took a jump to the unknown and a lot of courage to keep in my own path, but it was worth it."

Discussion

Even though the FL movement fascinates many teachers who are trying to change their learning culture to more student-centered learning approach teachers also have worries concerning students as well as the role of their own. The target of FL is closely linked to classical research questions about education but these questions are not easy for teachers to comprehend. Within this article both theoretical and emotional support will be offered to developers. Based on the data of this study, one of the problems in process of changing the learning culture is that the teacher is not strong enough to fight for the depression caused by the resistance of colleagues as well as the resistance of students and their parents. The teacher who flips his or her learning culture inevitably also questioning the learning culture of other teachers at the same school, too. At least in Finland the specific facebook group for flippers is a precious place for collaboration and peer support. It is the place to scare not only practical experiences but also pedagogical opinions. After strengthening our theoretical understanding about why FL seems to be functioning so well we can also understand better what is central in the approach we are trying to achieve by applying it. Pedagogical discussions where the critique is welcome will be needed to develop FL further to support the learning of individuals.

What is perhaps unexpected was that only one teacher out of 33 said that better learning results was her reason to flip. Mainly all of the scientific researches where learning results are compared between the flipped approach and more traditional way of teaching have ended up the conclusion that the learning results are lightly better in the flipped approach. However, the problem with these researches is that flipping is understood as a technical change in teaching, as a method, and not in a learning culture where the focus is on increasing engagement, autonomy and student centeredness.

Declaration of Conflicting Interests

The authors declared that they had no conflicts of interest with respect to their authorship or the publication of this article.

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References

Abeysekera, L., & Dawson, P. (2015). Motivation and cognitive load in the flipped classroom: Definition, rationale and a call for research. *Higher Education Research & Development, 34*(1), 1-14.

Anderson, R. (2007). Being a mathematics learner: Four faces of identity. *Mathematics Educator, 17(1)*, 7-14.

Ben-zvi, D., & Sfard, A. (2007). Ariadne's thread, Daedalus' wings and the learners' autonomy. *Education & Didactique, 1*, 117-134.

Bergmann, J., & Sams, A. (2012). *Flip your classroom. Reach every student in every class every day*. Washington, DC: International Society for Technology in Education.

Blanton, M. L., Westbrook, S., & Carter, G. (2005). Using Valsiner's zone theory to interpret teaching practices in mathematics and science classrooms. *Journal of Mathematics Teacher Education, 8(1)*, 5-33.

Bloom, B. S. (1968). Learning for mastery. *Evaluation Comment, 1(2)*, 1-12.

Carrol, J. (1963). A model of school learning. *Teachers College Recod, 64*, 723-733.

Chen, Y., Wang, Y., Kinshuk, & Chen, N. (2014). Is FLIP enough? or should we use the FLIPPED model instead? *Computers & Education, 79*, 16-27.

Davies, R., Dean, D., & Ball, N. (2013). Flipping the classroom and instructional technology integration in a college-level information systems spreadsheet course. *Educational Technology Research & Development, 61(4)*, 563-580.

Deci, E. L., & Ryan, R. M. (1985). *Intrinsic motivation and self-determination in human behavior*. New York: Springer.

Demetry, C. (2010). Work in progress - an innovation merging "classroom flip" and team-based learning. *40 Th ASEE/IEEE Frontiers in Education Conference* Washington. 26-27.

Dillenbourg, P. (1999). Introduction: What do you mean by "collaborative learning"? In P. Dillenbourg (Ed), *Collaborative learning: Cognitive and computational approaches* (1999th ed.). Amsterdam: Pergamon.

Foertsch, J., Moses, G., Strikwerda, J., & Litzkow, M. (2002). Reversing the lecture/homework paradigm using eTEACH web-based streaming video software. *Journal of Engineering Education, 91(3)*, 267-274.

Guskey, T. R., & Pigott, T. D. (1988). Research on group-based mastery learning programs: A meta-analysis. *Journal of Educational Research, 81(4)*

Holton, D., & Clarke, D. (2006). Scaffolding and metacognition. *International Journal of Mathematical Education in Science & Technology, 37(2)*, 127-143.

Johnson, G. B. (2013). *Student perceptions of the flipped classroom. Master of arts in the college of graduate studies. Educational technology*. The University of British Columbia.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *Elementary School Journal, 102(1)*

Koro, J. (1993). *Aikuinen oman oppimisen ohjaajana*. Jyväskylä: Jyväskylän yliopisto.

McCaslin, M. (2009). Co-regulation of student motivation and emergent identity. *Educational Psychologist, 44*, 137-146.

Moore, A. J., Gillett, M. R., & Steele, M. D. (2014). Fostering student engagement with the flip. *Mathematics Teacher, 107(6)*, 420-425.

Oerter, R. (1992). The zone of proximal development for learning and teaching. In F. Oser, A. Dick & J. Patry (Eds.), *Effective and responsible teaching: The new synthesis* (pp. 187-202). San Francisco: Jossey-Bass.

Ryan, R. M., & Deci, E. L. (2000). Intrinsic and extrinsic motivations: Classic definitions and new directions. *Contemporary Educational Psychology, 25(1)*, 54-67.

Stefanou, C. R., Perencevich, K. C., DiCintio, M., & Turner, J. C. (2004). Supporting autonomy in the classroom: Ways teachers encourage decision making and ownership. *Educational Psychologist, 39(2)*, 97-110.

Tangney, S. (2014). Student-centred learning: A humanist perspective. *Teaching in Higher Education, 19(3)*, 266-275.

Toivola, M., & Silfverberg, H. (2015). Flipped learning –approach in mathematics teaching – a theoretical point of view. *Proceedings of the Symposium of Finnish Mathematics and Science Education Research Association*, Oulu.

Valsiner, J. (Ed.). (1997). *Culture and the development of children's actions. A theory of human development* (Second Edition ed.). New York: John Wiley & Sons.

Volet, S., Vauras, M., & Salonen, P. (2009). Self- and social requlation in learning contexts: An integrative perspective. *Educational Psychologist, Vol. 44(4)*, 215-226.

Webb, N. M., Franke, M. L., De, T., Chan, A. G., Freund, D., Shein, P., & Melkonian, D. K. (2009). 'Explain to your partner': Teachers' instructional practices and students' dialogue in small groups. *Cambridge Journal of Education, 39(1)*, 49-70.

Yarbro, J., Arfstrom, K. M., McKnight, K. & McKnight, P. (2014). Extension of a review of flipped learning, *Flipped-learning network/Pearson/George Mason University*. Retrieved from http://flippedlearning.org/research

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article

A Journey through the geometrical alchemies of art: an example of constructivist teaching

Annarosa Serpe, Maria Giovanna Frassia

Abstract. *This paper illustrates a teaching setting which adopts a constructivist approach, and is centered on the combination of Mathematics and Art. The aim is to appeal to those minds that have a curiosity for a discipline which has long interacted with artistic expression, and to introduce new perspectives in the teaching of Mathematics in the light of the latest technological challenges. The trigger element - a decorative spiral frieze - has led to a process of Euclidean geometrical construction first with traditional tools, and then through the use of simple algorithms implemented in the MatCos programming environment. The drawing of the spiral curve represents a learning opportunity, a practice which combines analysis and synthesis, a journey between past and present which offers the student the opportunity to acquire some basic conceptual and instrumental implications in the age-old relationship between Mathematics and Art.*

Key words. *Archimedes' Spiral, Architectural Frieze, Constructivist Teaching Approach, Programming Environment.*

Sommario. *L'articolo propone una situazione didattica di taglio costruttivista imperniata sul binomio Matematica-Arte allo scopo di avvicinare le menti curiose a quella matematica che ha profondamente interagito con l'espressione artistica e, in virtù dalle nuove istanze poste dalle tecnologie, di introdurre nuove prospettive nell'insegnamento della Matematica. L'elemento attivatore, identificato nei fregi decorativi a forma di spirale, consente di mettere in atto un processo di costruzione geometrica euclidea prima con strumenti tradizionali, poi mediante semplici algoritmi implementati nell'ambiente di programmazione MatCos.*
Il disegno della curva spirale costituisce un'occasione conoscitiva, una pratica che racchiude in sé analisi e sintesi; un cammino tra passato e presente che offre allo studente l'opportunità di cogliere alcuni lineamenti concettuali e strumentali dell'atavico rapporto tra Matematica e Arte.

Parole chiave. *Ambiente di programmazione, Approccio didattico costruttivista, Fregio architettonico, Spirale di Archimede.*

Introduction

Today more than ever the teaching of Mathematics is motivated by the contribution it can give to the shaping of thought and personality as a whole. Teaching means taking into consideration not only the cultural role which pertains to Mathematics, but also the motivation which sustains this role in an ever changing society. Consequently, didactics needs to be practical and aim to recompose analytical knowledge, to re-affirm that all knowledge is part of a whole. Only in its entirety can it improve the understanding of reality. This calls for a re-visiting of traditional teaching, which is based on the transmission of notions, and which views learning as receptive elaboration of data, independent of others, and therefore solitary.

Constructing informed knowledge instead implies integrating different areas of interest which interact like mental models derived from cognitive science, co-operative learning and technological fields. The teaching of mathematics cannot neglect the synergy with other subjects. Research studies have indeed shown how the integration and harmonization of contents pertaining to different disciplines represent viable paths towards informed and lasting learning (Adelman, 1989; Check, 1992). Interdisciplinary connections in the educational process facilitate a more thorough comprehension of concepts, because the understanding can get to the root of notions and avoid 'airtight compartments' (Brooks & Brooks, 1993).

Interdisciplinary learning breaks the linear model of knowledge development and opens the door to constructivist didactics in which subjects display their full educational value within learning environments derived from the real world and based on cases, rather than on pre-determined instruction sequences (Wicklein & Schell, 1995). A constructivist type of didactics emphasizes the construction of knowledge and not its repetition, and facilitates reflection as well as reasoning, also through collaboration among peers (Jonassen et al., 1998).

Learning environments with the above-mentioned aims allow the students to experience a process of flexible re-adjustment of pre-existing knowledge according to the needs of the new educational experience.

In this paper, the authors put forward an example of an inter-disciplinary learning environment centred on the Mathematics-Art combination. A similar proposal might appeal to those minds that have a curiosity for a discipline which has long interacted with artistic expression, and could open up new perspectives in the teaching of Mathematics in the light of the latest technological challenges.

Theoretical Framework

In Italian higher secondary schools the connection of Mathematics with real life is an often stated truth. However, this connection is considered too difficult for the students to experience, except in a few standard cases mentioned in most textbooks. Some topics, for example, are avoided because they are difficult to manage during classes, especially as far as their visualization and graphic representation are concerned.

One example is given by algebraic curves. If we exclude the classic teaching paths on conics, other algebraic curves such as spirals, epicycloids, hypocycloids, etc., are usually ignored. This represents a gap in the learners' education because curves - geometrical objects *par excellence* - in the mathematical imagination play the delicate role of a border zone where different and sometimes opposed activities converge.

Curves offer a rich and partly unexplored field of study that can be discovered and rediscovered also at inter-disciplinary level, because they combine tangible objects and practical mechanisms with abstract mathematical thought; aptly included in classroom teaching, they help students to identify correlations between geometrical concepts, the mechanisms of technology and science constructions. As curves refer back to drawing, the latter becomes very important, located as it is between concrete reality and the abstract geometrical concept it represents; as a result, drawing is the basis of the dialectic process and relationship between the figural and the conceptual components (Mariotti, 1995).

A drawing represents an object observed and summarized in a few traits. It is a learning opportunity, a practice which combines analysis and synthesis. Indeed the tracing of a curve, defined as a locus of points which satisfy a given propriety, usually passes through the geometric construction of one of its points with classic tools like ruler and compasses.

However, today such tools can be integrated or substituted with recent educational software which allows the execution and manipulation of geometric constructions (Serpe, Frassia, 2015).

Such software programmes turn the classic traditional principles into information technology. Their use in the teaching - learning of Mathematics has been shown in the literature (Doerr, Zangor, 2000; Ruthven, Hennessy, 2002; Artigue et al., 2003; Lagrange et al., 2003; Monaghan, 2004; Lagrange, Ozdemir Erdogan, 2009; Dix et al., 2004; Guzman, Nussbaum, 2009; Collins et al., 1988; Costabile, Serpe, 2012 and many other).

The representation of a curve can also be seen as a visually beautiful shape, thereby connecting Mathematics and artistic expression. Mathematics and Art, expressive linguistic modes which are quite distant, are - despite appearances - connected to each other, because both subjects involve the imagination and creativity.

The present work fits into this theoretical framework, and puts forward a constructivist teaching practice. Here visual art and geometry meet in a technological setting through the practice of programming, whose pedagogic and formative value is known in the literature (Pea, Kurland, 1984; Kurland et al., 1989; Oprea, 1988; Serpe, 2006 and many other).

The trigger element was identified as a decorative spiral frieze, which led to a process of Euclidean geometrical construction first with traditional tools, and after through simple algorithms implemented in the MatCos[1] programming environment.

A similar path uses design as a tool for formal analysis between past and present and joins a tangible object to abstract mathematic thought. It is a learning path in which students realize how mathematical reasoning and art share analogies, how shapes derived from empirical solutions can stand tests with deeper meanings, reasons and evaluations.

Methodology and Design Activity

The learning environment was set up with the specific aim of guiding students to the understanding of the relationship between concrete versus abstract, staying away from purely theoretical mathematical concepts. The activity begins with the analysis of the decorative frieze on the façade of the *San Matteo al Cassaro* church in Palermo (Italy), and later moves on to the geometric investigation which has the construction of concepts and meanings as its objective.

The design implies the study of the geometrical method for tracing Archimedes' spiral in Euclidean terms using drawing tools like ruler, set square and compasses and later also with a computer as a programming tool. We chose to use computer programming because it is a constructive and creative activity, which reinforces the acquisition of concepts as well as abstraction skills.

The construction of the Euclidean curve represents a strategic phase from the teaching point of view because it helps the learner to perceive the complexity of the whole, which starts from 'simple and evident' and arrives at the 'complex and not evident'. Geometric construction indeed helps students to start this process which will take them from easy to difficult in a tangible, critical and rigorous way.

The re-interpretation of data through information technology tools enables the learner to turn the meaning of a representation into its construction (filling the gap between experience and creation) through a model which shows and communicates geometrical synthesis.

Mathematical modelling has expressive and evocative potential from the metric and formal point of view; the virtual tracing of a curve through simple algorithms adds value to the experience:

«*The interaction between a learner and a computer is based on a symbolic interpretation and computation of the learner input, and the feedback of the environment is provided in the proper register allowing its reading as a mathematical phenomenon*». (Balacheff & Kaput, 1996, p.470).

Such methodology avoids the traditional pitfalls of Mathematics classes, and gives students a true chance to stretch their views.

The geometric reading of a frieze: Archimedes' spiral

Identifying the trigger element: the decorative spiral frieze

The teaching sequence begins with a guided virtual tour of the city of Palermo where the students are attend one of the first two years of higher secondary school.

With the help of their Drawing and Art History teacher, they discover the city's artistic and architectural heritage. During the virtual tour the students are asked to pay special attention to the decorative elements on the façades of buildings. This way the students have the opportunity to observe and recognize some special geometrical shapes in the architectural friezes. During the observation phase an extraordinarily beautiful shape - which develops with an enveloping movement - is identified in the large coils found on the façade of the *San Matteo al Cassaro* church[2] (see Figure 1).

Fig.1 - The façade of the *San Matteo a Cassaro* church

First the Drawing and Art History teacher sets up a discussion which highlights how this enveloping shape - called spiral - is a figure created on the basis of a more or less complex plan to transfer information about real or imaginary objects, concepts and emotions; so the matter is worth investigating.

Here the Mathematics teacher steps in, in order to take the work further, as the students progress from their experience of discovery to research pathways where they find out more, and learn about, the architectural frieze (see Figure 2).

Fig.2 – Detail of a large spiral coil

Developing the empirical situation: geometrical reading of the frieze

Developing the empirical situation through a process of mathematical enables the learners to make a considerable leap in abstraction: the concise and objective description of the curve requires the introduction of concepts and tools that are acquired and tested in the model study phase. Later the evaluation of the model allows to perfect the spiral construction algorithm, as well as a reflection on the theoretical aspect and further needs analysis. For all the above reasons, the mathematical modelling process consists of three steps.

In the first step the students draw a curve using the traditional tools, ruler and compasses, and later proceed to use the virtual tools through the MatCos[3] The second step is very relevant from the point of view of abstraction because it requires a refining of the previous construction process through the construction of the 'for' cycle or iteration.

The last step has the aim of further perfecting the construction of the curve. Such a highly educational process trains the students in the appreciation of the potential of the language of Mathematics and at the same time offers them a reading awareness of theories.

The steps are described in detail below.

<u>Step 1</u>

The class is divided in groups and each group receives the tools and material for the task (sheet of paper, ruler, compasses, pencils and rubbers); afterwards, the teacher sets the following task:

1. Construct the square *ABCD*;
2. Determine the median points (*E*, *F*, *G*, *H*) of the sides of the square;
3. Construct the rays (*e*, *h*, *g*, *f*) with origin the median points of the square and passing by one of the vertex of the square and belonging to the side on which the median point itself lies (*anticlockwise direction*);
4. Construct the circumference with centre *A* and passing by point *B*;

5. Determine the point of intersection *I* between the circumference and the ray with origin the median point of side *AB* and passing by point *A*;
6. Draw the arc *p* with centre *A* and points *B* and *I* as extremes;
7. Repeat from 5 a 7:
   - I. Construct the circumference *CC* with centre *A*, *B*, *C*, *D* (in this order) and radius the distance between the vertex of the square and the point of intersection between the old circumference and the rays *e, h, g, f* (in this order);
   - II. Construct the arc with centre in the vertex of the square.

The above-mentioned geometric construction is executed four times in order to complete the first turn of the spiral; after that the students should implement it in the MatCos[4] programming environment (see Fig.3).

/*Programme Code MCS1*/

```
A=point; B=point; l=segment(A,B);
p1=perpendicular(l,A); D=EndPoint(p1,/2); p2=perpendicular(l,B);
C=EndPoint(p2,2); s=segment(C,D);
E=Midpoint(l); F=Midpoint(p1); G=Midpoint(s);H=Midpoint(p2);
e=ray(E,A); f=ray(F,D); g=ray(G,C); h=ray(H,B);
PenColor(0,128,0);PenWidth(5);
r1=Distance(A,B); cc=circ(A,r1);
I1=intersection(cc,e); p1=arc(A,B,I1,ANTICLOCKWISE);
r2=distance(B,I1); cc=circ(B,r2);
I2=Intersection(cc,h); p2=arc(B,I1,I2,ANTICLOCKWISE);
delete(cc);
r3=distance(C,I2); cc=circ(C,r3);
I3=intersection(cc,g); p3=arc(C,I2,I3,ANTICLOCKWISE);
delete(cc);
r4=distance(D,I3); cc=circ(D,r4);
I4=intersection(cc,f); p4=arc(D,I3,I4,ANTICLOCKWISE);
delete(cc);
```

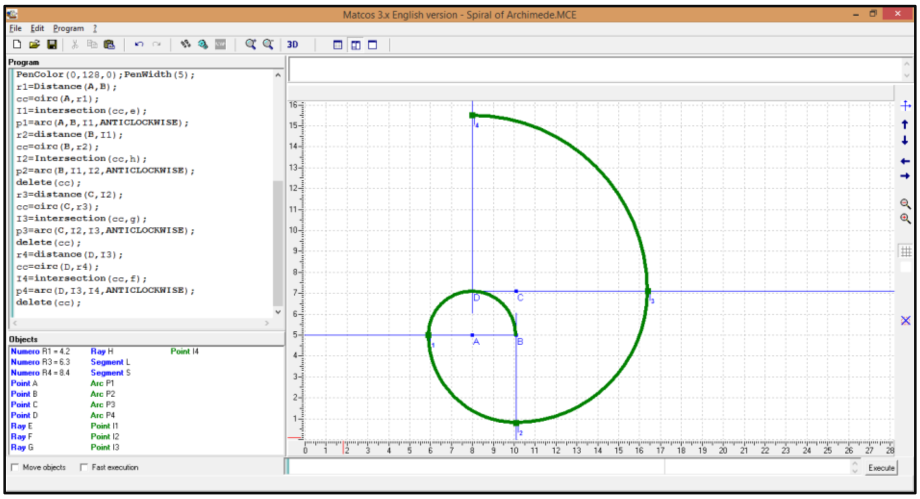

Fig.3-Output of the first coil

Materials, tools and task get the students going until they achieve the geometrical reading of the decorative frieze. The change from traditional to virtual instruments adds value to the learning path: the MatCos programming environment is used just like a real drawing pad, but it allows a more accurate syntax of the Euclidean register.

Step 2

The construction algorithm enables the students to better learn the characteristics of the curve. However, its repetitive execution could be boring if one wanted to reproduce the decorative frieze fairly accurately.

From this derives the need to shorten the sequence of repetition (the drawing of the joined arcs). The previous construction algorithm can be refined making use of the 'for' option to draw the joined arcs. The new formalized algorithm (see Fig.4) is:

1. Draw two points *A* and *B*;
2. Construct the square with side *A* and *B*;
3. Determine the median points of the sides of the square;
4. Construct the rays with origin the median points of the square and passing by one of the vertexes of the square and pertaining to the side on which the median point lies (*anticlockwise direction*);
5. Construct the circumference with centre *A* and passing by point *B;*
6. Determine the point of intersection *I* between the circumference and the ray with origin the median point of side *AB* and passing by *A*;
7. Trace the arc *p* with centre *A* and extremes the points *B* and *I*;
8. Cycle for the construction of the joined arcs.

```
/*Programme Code MCS2*/
n=readnumber;
A=point; B=point; l=segment(A,B);
p1=perpendicular(l,A); D=Endpoint(p1,2);
p2=perpendicular(l,B); C=Endpoint(p2,2);
s=segment(C,D); polygon(A,B,C,D);
E=Midpoint(l); F=Midpoint(p1); G=Midpoint(s); H=Midpoint(p2);
e=ray(E,A); f=ray(F,D); g=ray(G,C); h=ray(H,B);
PenColor(0,128,0);PenWidth(5);
r=distance(A,B); cc=circ(A,r);
I1=intersection(cc,e); arc(A,B,I1,ANTICLOCKWISE);
delete(cc);
for(i from 1 to n) do;
P=SelectObject; Q=SelectVertex;r=distance(P,Q);
semirett=SelectObject; cc=circ(Q,r);
I1=intersection(cc,semirett); arc(Q,P,I1,ANTICLOCKWISE);
delete(cc);
end;
delete(semirett,I1,P,Q);
```

Refining the previous process of construction is relevant because the students are required to make a considerable abstractive leap: what is difficult is the identification of the construction, within the algorithm, which enables the repetition of an operation (construction of joined arcs) as long as a certain condition is true.

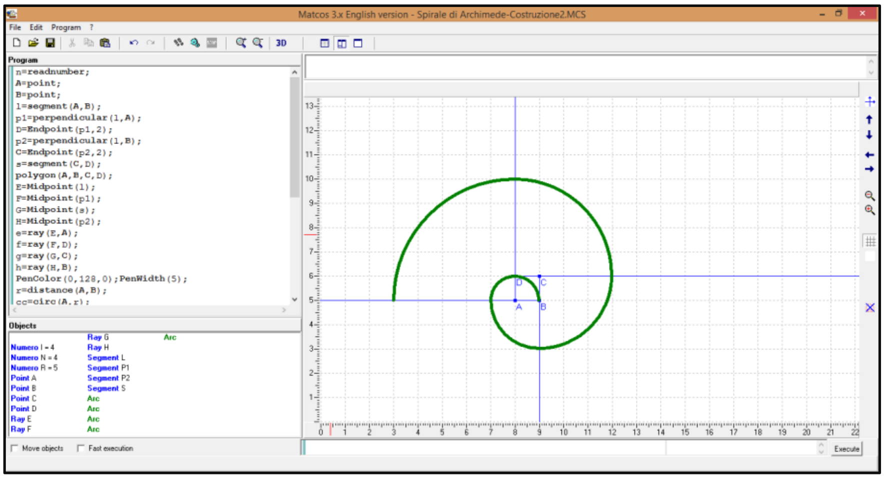

Fig.4-Output of the refined algorithm

<u>Step 3</u>

At the end of the second phase it is important to reflect on possible further improvements; once again the students are encouraged to make another abstraction leap. Careful observation leads the participants to notice that the second extreme of each joined arc can be obtained from the rotation of the first extreme by 90°; that is why the construction of the rays starting from the vertices of the square, of the circumference and of the intersection point between the half-line and the circumference becomes superfluous. Consequently, the final formalized algorithm (see Figure 5) is as follows:

1. Draw two points *A* and *B*;
2. Draw point *A'*, obtained from the rotation of *A* with respect to centre *B* by a 90° angle anticlockwise;
3. Construction of the arc with centre *B* and extremes *A* and *A'*;
4. Cycle for the construction of the joined spiral arcs with appropriate choise of the required objects at point 2.
5. /* Programme Code MCS3*/

```
n=readnumber;
A=point; B=point; l=segment(A,B);
p1=perpendicular(l,A); D=Endpoint(p1,2);
p2=perpendicular(l,B); C=Endpoint(p2,2);
s=segment(C,D); polygon(A,B,C,D);
PenColor(0,128,0);PenWidth(5);
P=SelectObject; Q=SelectVertex;
PP=rotate(P,Q,90,ANTICLOCKWISE); arc(Q,P,PP,ANTICLOCKWISE);
for(i from 1 to n) do;
P=SelectObject; Q=SelectVertex;
PP=rotate(P,Q,90,ANTICLOCKWISE);
arc(Q,P,PP,ANTICLOCKWISE);
end;
delete(A,B,C,D,P,PP,Q);
```

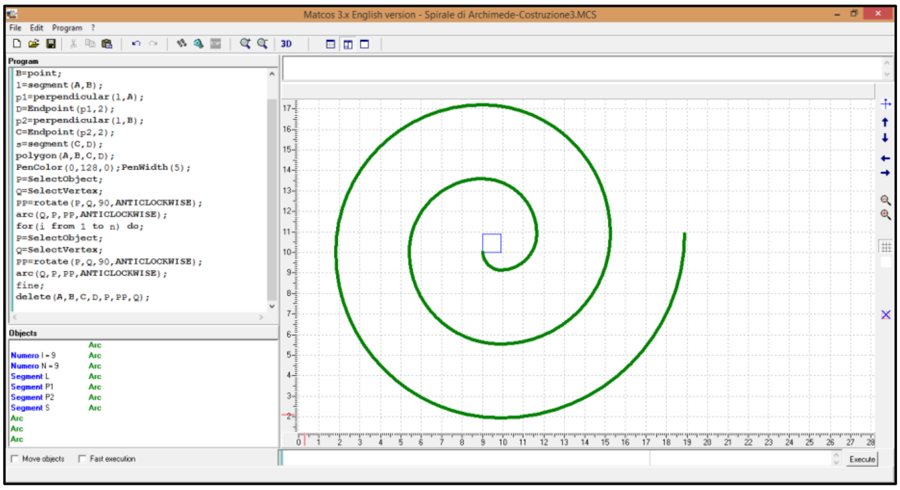

Fig.5 - Final Output of Archimedes' spiral

Discussion

The topic unfolds around the enveloping movement of the spiral geometrical shape which represents a recurring theme in Art, and in the synchronic interpretation of the geometrical connections to be identified.

The constructivist approach allows the students to work on evolving shapes. In the first step we noticed that the students showed different degrees of manual skills when drawing with traditional tools like ruler and compasses. This is almost certainly due to the process of idealization where material instruments become abstract objects. That is why we got the learners to spend quite some time on the virtual manipulation of geometrical shapes to increase their awareness of them. The implementation of the first construction algorithm for the curve, repeated several times, facilitated the understanding and use of the geometrical objects; however, at the same time the need to shorten the repetition sequence for the construction of the joined arcs emerged for the first time. At this point along the activity interaction among the students in their groups had considerably improved. They developed a spirit of co-operation and solidarity among peers far greater than previously.

This is the result of group work, where everyone feels the need to be accepted so everyone gets involved. Only two groups worked on the improvement of the algorithm, and they immediately shared their findings with the rest of the class without any intervention by the teacher. This 'coaching between peers' is a valid learning strategy because skills and knowledge are acquired through direct personal effort and emotional involvement.

Conclusion

One of the main difficulties in teaching and communicating Mathematics is its abstract nature. In order to make it concrete and more easily understandable to students it is essential to use different strategies that can reveal the crucial role Mathematical reasoning plays in our lives, and may uncover the beauty of the subject. The setting up of a similar learning environment is an attempt in this direction, and aims to show how artistic expression and mathematics, apparently pertaining to very distant spheres of knowledge, can indeed be connected because they both require imagination and creativity. Combining these two expressive forms into a teaching situation has the effect of raising interest for Mathematics and awakening mathematical aptitude.

It is a motivating initiative, which provides stimulation and offers the tools for different learning styles. It also encourages self-learning through creativity and peer-teaching and - starting from previous notions - helps the students to interpret their own experiences and construct more complex and formally articulated knowledge and competences.

To conclude, this constructivist teaching initiative is to be intended as a fascinating journey through Art's geometrical alchemies which will break down the cultural barriers that have long existed between the two disciplines in school education, while at the same time recovering classical notions through the use of methods which are up-to-date and which can captivate the students' attention.

Declaration of Conflicting Interests

The authors declared that they had no conflicts of interest with respect to their authorship or the publication of this article.

Repository of materials

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http://www.edimast.it/J/20160201/02490260SE/

Note

1. The MatCos programming environment, now in its 3rd version, was devised and created on the following pedagogic-educational paradigm:
   - Introducing the learners to computer programming using mathematical concepts suitable for their age group;
   - helping students to learn mathematical concepts and methods exploiting the potential of computers.

   For specific educational aspects and technical requirements, you can visit the Web Site Url: http://cird.unical.it/.
2. The Church of *San Matteo al Cassaro* is a place of worship in the historic centre of Palermo that gives out onto the street *del Cassaro* (now Vittorio Emanuele Avenue). In the three floors of the principal façade, completed in 1662 and richly decorated, three niches were carved out which house the statues of *La Vergine* in the middle, *San Matteo Apostolo ed Evangelista* on the left and *San Mattia Apostolo* on the right. Above the portal, on the second floor an elaborate cornice separates the two levels and on the external sides features large, spiral, connecting coils which can be found also on the third floor. The church was built by the architects and sculptors Carlo D'Aprile, and Gaspare Guercio who followed in Vincenzo Guercio's family footsteps.
3. The students know the MatCos programming environment quite well because it is used for the Mathematics curriculum activities.

References

Adelman N. (1989). *The case for integrating academic and vocational education*. Washington: DC: Policy Studies Associates, Inc.

Artigue M., Drijvers P., Lagrange, J.B., Mariotti M.A., and Ruthven K. (2009). Technologies Numériques dans l'enseignement des mathématiques, oùen est-on dans les recherches et dans leur intégration? In C. Ouvrier-Buffet, M.J. Perrin-Glorian (Eds.), *Approches plurielles en didactique des mathématiques: Apprendre à faire des mathématiques du primaire au supérieur: quoi de neuf?* Paris: Université Paris

Diderot Paris 7, pp.85–207.

Balacheff N., Kaput J.J. (1996). Computer-Based Learning Environments in Mathematics. In A.J. Bishop et al. (Eds.), *International Handbook of mathematics education*. Dordrecht, Netherlands: Kluver Accademic Pubblishers, pp.469-501.

Beane J. (1996). On the shoulders of giants! The case for curriculum integration. *Middle School Journal*, 25, 6–11.

Brooks J.G., Brooks, M.G. (1993). *In search of understanding: the case for constructivist classrooms*. Alexandria, VA: Association for Supervision and Curriculum Development. http://www.ascd.org

Cheek D. (1992). *Thinking constructively about science, technology and society education*. Albany NY: State University of New York Press.

Collins A., Brown J.S. (1988). The Computer as tool of learning trough reflection. In H. Mandl, A. Lesgold (Eds), *Learning Issue for Intelligent Tutoring Systems*. New York: Springer, pp.1-8.

Costabile F.A., Serpe A. (2012). MatCos: A Programming Environment for Mathematics Education. In G. Lee (Eds.), *Proceedings of the International Conference on Computer and Advanced Technology in Education* (Vol. 859971, pp. 25-30). Beijing, China, ICCATE 2011.New York: ASME Press: New York. http://asmedl.org/ebooks/asme/asme_press/859971

Dix A., Finlay J., Abowd, G.D., and Beale R. (2004). *Human-Computer Interaction*. (3rd ed.). Harlow, England: Pearson Education.

Doerr H.M., Zangor R. (2000). Creating meaning for and with the graphing calculator. *Educational Studies in Mathematics*, *41*, 143–163.

Jonassen, D.H., Peck, K L., Wilson, B.G., and Pfeiffer, W.S. (1998). *Learning with technology: A constructivist perspective*. Upper Saddle River, New York: Prentice-Hall.

Guzman A., Nussbaum M. (2009). Teaching Competencies for Technology Integration in the classroom. *Journal of Computer Assisted Learning*, *25*, 453-469.

Kurland D.M., Pea R.D., Clement C.and Mawby R. (1989).A study of the development of programming ability and thinking skills in high school students. *In* E. Soloway, C.S. *James* (Eds.), *Studying the Novice programmers (pp.* 83-109*).* Hillsdale, NJ: Lawrence Erlbaum Associates.

Lagrange J.B., Artigue M., Laborde C. and Trouche L. (2003). Technology and mathematics education: A multidimensional study of the evolution of research and innovation. In A.J. Bishop, M.A. Clements, C.Keitel, J. Kilpatrick, and K.S.Leung (Eds.), *Second International Handbook of Mathematics Education*. Dordrecht: Kluwer, pp.237-269.

Lagrange J.B., Ozdemir Erdogan E. (2009). Teachers' emergent goals in spreadsheet-based lessons: analyzing the complexity of technology integration. *Educational Studies in Mathematics*, *71*(1), 65-84.

Mariotti M.A.(2005). *La geometria in classe. Riflessioni sull'insegnamento e apprendimento della geometria.* Pitagora Editrice Bologna, pp.33-34.

Monaghan J. (2004). Teachers' activities in technology-based mathematics lessons. *International Journal of Computer for Mathematical Learning*, *9*, 327–357.

Oprea J. M. (1988).Computer programming and mathematical thinking. *Journal of Mathematical Behavior*, *7*(2), 175-190.

Pea R.D., Kurland D.M. (1984). On the Cognitive Effects of Learning Computer Programming. *New Ideas in Psychology*, *2*(2), 137-168.

Ruthven K., Hennessy S. (2002). A practitioner model of the use of computer-based tools and resources to support mathematics teaching and learning. *Educational Studies in Mathematics*, *49*(1), 47-88.

Serpe A. (2006). La programmazione per il rinnovamento dell'insegnamento della matematica. In C. Di Stefano C. (ed), Atti del convegno nazionale *Sul rinnovamento per l'insegnamento della matematica*, (pp.33-48). Gela (CL), 5-7 ottobre 2006, Ghisetti&Corvi Editore,

Serpe A. Frassia M.G. (2015). The deltoid as envelope of line in high school: A constructive approach in the classroom. In C. Sabena, & B. Di Paola (Eds.), *Proceedings of CIEAEM 67: Teaching and Learning Mathematics: Resources and obstacles. Quaderni di Ricerca in didattica (Mathematics), N.25* Supplemento n.2 Palermo: G.R.I.M. Press University of Palermo (Italy), pp.549-557

Wicklein R.C., Schell J.W. (1995). Case studies of multidisciplinary approaches to integrating mathematics, science, and technology education. *Journal of Technology Education*, *6*(2), 59-76.

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article

Practices and Possibilities in Nepalese Mathematics Education

Toyanath Sharma

Abstract. *Nepal is a developing country and it has innumerous challenges to accomplish. As a mathematics educationist, we are facing huge amount of challenges. In this paper, I am trying to fill the gap between international context of mathematics education and our way of practices. We are in a postmodern era, according to time and development, yet we are still practicing our conventional way of teaching and learning in mathematics. From the view of critical mathematics pedagogy, we have many ways to transform our traditional practices into the modern mechanism. We (Nepalese) are always oppressed by our culture, socio-political condition, our beliefs and foundation, civilization, religious practices, information and technology, economic status and educational practices. Despite of having such tribulations we are trying to change our society through our educational practices. Mathematics is considered as a key to triumph and it is essential in our daily life routine. For that, this small research can work as a bridge between our recent context and emergent issues in Mathematics Education.*

Key words. *Critical Math Pedagogy, Contextual Mathematics, Cultural Mathematics and Mathematics Education*

Sommario. *In un Paese in via di sviluppo come il Nepal, gli insegnanti di matematica sono chiamati a fronteggiare numerose sfide. In questo articolo cercherò di colmare la distanza tra il contesto internazionale in cui vive l'insegnamento della Matematica e le pratiche in uso nel nostro Paese. Viviamo in un'era postmoderna rispetto ai tempi e agli sviluppi, tuttavia ci si serve ancora di pratiche convenzionali nell'insegnamento e nell'apprendimento della Matematica. Secondo un approccio pedagogico critico, si dispongono di molti modi per trasformare le pratiche esistenti relative alla Matematica in meccanismi moderni. I Nepalesi risultano oppressi dalla loro cultura, condizioni sociali e politiche, le credenze, la civilizzazione, le pratiche religiose, l'informazione e la tecnologia, stato economico e pratiche educative. Si cerca attraverso pratiche educative di liberarsi da tali oppressioni e in questo senso la Matematica viene vista come un modo per trionfare nella routine della vita quotidiana.*

Parole chiave. *Critical Math Pedagogy, Contextual Mathematics, Cultural Mathematics and Mathematics Education.*

Introduction

Our country is in the political reformed stage and it still needs a great amount of effort to face the challenges for the sole transformation. To heal this burning situation "*Mathematics is … a purposeful activity to deal with social, political and economic goals and constraints. It is not value-free or culturally-neutral*" (Almeida, 2010). Therefore, it can be a source for alteration. Transformation is a continuous process, it means, the way is long for our educational reform. This reformation program insists on the essentiality of two laddering steps and they are: (i) our belief system and (ii) physical structure and facilities. We have a traditional way of mathematics teaching-learning process and modern expansionary education is daily challenging us. On the one hand, Mathematics education could mean empowerment, but on the other hand it denotes suppression as well. It could mean inclusion but at the same time it means exclusion and discrimination: "Mathematics is not only an impenetrable mystery to many, but more than any other subject, has been cast in the role of an 'objective' judge, in order to decide who in the society 'can' and who 'cannot'. Therefore It serves as the gatekeeper to participation in the decision making processes of society. To deny some access to participation in mathematics is then also to determine, a priori, who will move ahead and who will stay behind." (Volmink, 1994). Our mathematics curriculum is somehow good to this society but our instructional activities may not. We still are following conventional "chalk and talk" method in mathematics classroom.

Most of Nepalese teachers are unable to address our students' voice. Vast majority of mathematics teachers are still teaching skill and drill in ways that serve only a select set of students (Confrey & Kazak, 2006).There are bags of things to execute. Our physical structure as well needs to modify. International students are already befriended with internet since decades but our students still do not know about computer; they are hanging on white papers messed up with black paint. Large numbers of students, especially from rural areas, are blind and blank even about TV. Information technology is beyond our access. Few city students are receiving those kind of facilities but rest of the students is living in the light of lamp. Critical mathematics education can be a route for the renovation of our society and structure. Critical mathematics talks about mathematics associating with power relations. knowledge and understanding of mathematics from the perspective of critical pedagogy is understood as a means for student (and teacher) self-empowerment to organize and reorganize interpretations of social institutions and traditions, and to develop proposals for more just and equitable social and political reform (Skovsmose, 1994).

Critical Mathematics Education in Nepal

Our society is a complex one. We are living in the 21st century; it has many merits and challenges too. Survival with dignity is the universal problem, to be faced by humankind. In these days mathematicians are severely involved to search out all the issues that are affecting the society. But we learn, through History, that the technological, industrial, military, economic and political complexes have developed thanks to mathematical instruments (Ernest, 2012). Critical mathematics education refers to a set of concerns or principles that work as catalysts for re-conceiving and redesigning the lived experience of school mathematics. These concerns or principles are meant to target issues of political agency in society through an examination of mathematics education. (Freitas, 2008). Further, Freire (1970/1998) advocated for a critical pedagogy that was grounded in the "present, existential, concrete situation" (76) whereby teaching might begin with the lived experiences of students, accessing their emotional and ethical ties to the situations in which they struggled for voice and equity (as cited in Freitas, 2008). In a Nepalese context, we are teaching textbook or content which is deploring the expansion of knowledge is increasing repeatedly. So, Nepalese mathematics teacher need to be critical, they need to reflect their classroom practices.

Self-questioning and self-actualization is necessary requirement for modern mathematics teachers. Mathematics textbooks are written only for the sole purpose of the business. They care less about child psychology. Textbooks too often specify the appropriate tool to be used for the given problem, and leave out the crucial ethical moment of reflecting on whether the means suit the ends (Freitas, 2008). Therefore, as a critical tutor researcher, I would like to draw the attention of policy makers and mathematics teachers on some relevant questions: *Are you teaching students? Or text book? Are you passionate about teaching? Why do they teach for money or profession? Do they love mathematics? Do they love children? Do they concern about our future? Are they developing child friendly environment in their classroom? How can we make our classroom effective by using our own available resources?* I think these questions are valuable. Skovsmose and Borba (2004) are careful to suggest that the critical approach must always tend to the "what if not" of school mathematics, that it must investigate the possible - think the otherwise - and explore "what could be." (p. 211). Moreover, they argue that researchers and educators must imagine alternatives about the mathematics education, which would be more inclusive, more playful and more relevant which finally assist to heal the current obstacles and problems by generating visions or descriptions with high activation and creation. The approach is profoundly hopeful and imaginative and offers educators a positive (and critical) mean for professional development. "*It confronts what is the case with what is not the case but what could become the case.*" (Skovsmose & Borba, 2004, p. 214). At least, we can give our own effort to initiate a new journey from our base. We are also educating our children with the attempt to make them more familiar about mathematical knowledge and its significance in their future life. Therefore, mathematics teachers play vital task to extend mathematical attitudes of children.

Developing teacher's attitude towards Mathematics Education

Before starting a career as a mathematics educator, a teacher needs to know about the importance and impact of mathematics on our society. What is more important: quantity of information or quality of knowledge? (Almeida, 2010). Mathematics education can support the development of an ideology of certainty (Borba and Skovsmose, 1997). Whenever we use absolute language, it becomes a little bit difficult to get its real life applications. Mathematics educators need to develop positive attitude towards mathematics so that the Student can think alternatively.

They will teach students to contextualize their solutions in terms of their own authority and social position presenting applications of problems in terms of hesitation, hedging, and ambiguity, and ask students to model their responses on the same language. Reflecting on the language of uncertainty allows us to reflect on the ethical dimension of our problem solving, to reflect on the implication of our proposed solutions (Freitas, 2008). Nepalese Mathematics educators should concern about the teaching methods that we are promoting; attitudes that we are developing; resources that we are using; beliefs that we are having and future that we are designing. Therefore, mathematics pedagogy can be an appropriate pedagogy for us to solve all these critical issues. Critical mathematics pedagogy is a tool to teach mathematics for social justice. Teaching mathematics for social justice has two dialectically related sets of pedagogical goals: one set focuses on social justice and the other set focuses on mathematics (Gutstein, 2006). Mathematics education is a weapon to give voice to oppressed people into the mainstream development. I mean critical mathematics pedagogy can be an influential tool for modern society. To achieve this goal we must make of our teacher a critical teacher/ researcher or reflective practitioner.

Situating Nepalese Mathematics Education in a Global Context

The World Civilization is anchored in Mathematics. No one disagrees that Mathematics is the dorsal spine of the modern world. But this leads to focus the concerns about the future on Mathematics (D'Ambrosio, 2010). Real world is a small global society being attached with each human conscience and everything is interrelated, so mathematics and our societies are also interconnected. Mathematics always helps society for problem solving (Keitel, 2009). Mathematics education is rarely referred to as a "weapon in the struggle" for social justice and equity, but nothing in principle prevents us from enacting such a vision. And, there are increasing numbers of teachers and mathematics educators engaged in such practices (see, for example, Gutstein & Peterson, 2005). Mathematics Education is a wealthy research area. Its importance for Education in general is unquestionable. Mathematics Education is remarkably interdisciplinary as an area for research. It relies on research in various disciplines, particularly in cultural studies and cognitive sciences (D'Ambrosio, 2010). We understand the nature of lived experience as a mix of chaos and order, thereby giving rise to a multi-epistemic space in which to account for the artful world of experience (Mahony, 1998).

The position that academic mathematicians (are forced to) adopt now for their discipline is that it involves *mathematiziation: to mathematise* is to search for and describe patterns, to generalize, to make predictions, to revise conjectures and to prove. That is, "mathematics is what mathematicians do" (Grunetti and Rogers, 2000). To promote creativity, helping people to fulfill their potentials and raise to the highest their capability, but being careful not to promote docile citizens. We do not want our students to become citizens who obey and accept rules and codes, which violate human dignity (D'Ambrosio, 2010). Further, he added that education should promote citizenship teaching values and the importance of rights and responsibilities in the society, but it is important to be careful to avoid any form of irresponsible creativity. We do not desire our student to become brilliant scientists discovering the weapons and instruments of suppression and inequity. As a mathematics teacher, I am trying to incorporate the global views of mathematics education in classroom practice. When I was a traditional mathematics teacher, I was teaching textbook, which was not proficient to enhance the creativity and globalize the brain of the students. However, the pathetic thing is that, at the same time, I was not expecting my student to be a traditional learner. Is it possible? I started raising finger towards myself with lots of queries that were teasing my brain. I have a dream to contribute in the activity-based mathematics teaching, so that students can have easy access to its concept without any anxious feeling. The dream is a way of knowing, and it stimulates responses and attempts to understand it that collaborates with other modes of cognition (McNiff, 2008, as cited in Sharma 2012). As a critical teacher researcher, I will try to develop conceptual mathematics teaching process in my class. From now on, I will try to articulate my experience as a part of my reflecting journey of mathematics teacher, student and a novice researcher (Sharma, 2012). Our school mathematics needs a vision concerning with mathematical literary genre which would be supportive to supply the skills and talent that is crucial for local and global citizenship. That is why; we need to develop skills and abilities to perform a two-way border crossing.

Conclusion

Mathematics educators of Nepal do have scores of problems teaching learning process. Still, we do not have any epitomic or marvellous results of educational deeds. Our educational structure is still not supporting us, according to the global context, but it is our worst plight that we have to struggle with modern society. Even now, we are not thinking about any foreseeable event. Critical self-reflection is the most powerful instrument for mathematics educators. We can initiate with the available resources and our current practices. We must think positive towards our profession and should change our attitude towards mathematics education. There is a saying that ‘Charity begins at home’. Therefore, every mathematics educator needs to give his/her effort to change within him/herself. Our knowledge, anyhow, is more or less similar to global practice but our practices are different, so attitude is the key to change our society through mathematics education. As the mathematics is everywhere, there are the golden chances for mathematics educators to be the leaders of the modern society.

Declaration of Conflicting Interests

The authors declared that they had no conflicts of interest with respect to their authorship or the publication of this article.

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References

Borba, M. & Skovsmose, O. (1997). The Ideology of Certainty in Mathematics Education. *For the Learning of Mathematics*, 17(3), 17-23.

Confrey, J. & Kazak, S. (2006). A Thirty Year Reflection on Constructivism in Mathematics Education. In A. Gutierrez & P. Boero (Eds.). *Handbook of Research on the Psychology of Mathematics Education*: *Past, Presentand Future*. Rotterdam, ND: Sense Publishing. 367-402.

Almeida, D.F. (2010). *Are There Viable Connections Between Mathematics, Mathematical Proof And Democracy?*. University of Exeter and Canterbury Christ Church University, UK

D’Ambrosio, U. (2010). *Ethnomathematics*: *A Response to the Changing Role of Mathematics in Society*. State University of Campinas, Brasil.

Ernest, P. (2012). *The Scope and Limits of Critical Mathematics Education*. University of Exeter, UK.

Freitas, E. (2008). Critical Mathematics Education: Recognizing the Ethical Dimension of Problem Solving. *International Electronic Journal of Mathematics Education*. Vol. (3)2. www.iejme.com.

Grunnetti, L. & Rogers, L. (2000). *Philosophical, multicultural and interdisciplinary issues*, in Fauvel and van Mannen, History in Mathematics Education, Kluwer.

Gutstein, E. (2006). *Possibilities and Challenges in Teaching Mathematics for Social Justice*. University of Illinois-Chicago.

Gutstein, E., & Peterson, B. (Eds.). (2005). *Rethinking mathematics: Teaching social justice by the numbers*. Milwaukee, WI: Rethinking Schools, Ltd.

Keitel, C. (2009). *Mathematics, Knowledge and Political power*. Sense Publishers.

Mahony, W. K. (1998). *The artful universe: An introduction to the Vedic religious imagination*. New York: SUNY Press.

Sharma, T. (2012). *Becoming a 'Good' Mathematics Teacher: An Epic Journey Through Different Mathematical Terrains*. Unpublished dissertation, Kathmandu University. Nepal.

Skovsmose, O. & Borba, M. (2004). Research methodology and critical mathematics education. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology*. New York, NY: Kluwer Academic Publishers.

Smitherman, S.E. (2006). *Reflections on Teaching a Mathematics Education Course*. Unpublished dissertation. Louisiana State University. Texas.

Volmink, J. (1994). Mathematics by All. In S. Lerman (Ed.), *Cultural Perspectives on the Mathematics Classroom*, pp. 51-67. Dordrecht: Kluwer.

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software

Authoring software and JClic – Part III

Giorgio Musilli

Abstract. *JClic: activities and advanced procedures; 0.3.1.0 version.*

Key words. *authoring software, jclic, learning objects, educational software.*

Sommario. *Le attività di JClic; configurazioni e procedure avanzate; novità nella versione 0.3.1.0.*

Parole chiave. *software autore, jclic, oggetti di apprendimento, software didattici.*

Introduzione

In questo contributo verranno presentate le attività che è possibile costruire con JClic, le configurazioni avanzate del programma e le novità della versione 0.3.1.0. Tutte le attività di JClic possono essere eseguite all'interno delle aree "*Attività*" e "*Sequenze*" cliccando sul triangolo verde con la punta rivolta a destra (vedi Fig. 1). In esecuzione, con lo stile di default, si vedono:

1) In alto, il nome del progetto;
2) Al centro, la finestra con il pannello di gioco;
3) In basso a sinistra, le due frecce di spostamento, la bandierina verde per ripetere l'esercizio, i tasti di aiuto e di informazione;
4) In basso al centro, i messaggi (iniziali, finali, d'errore);
5) In basso a destra, il punteggio, i tentativi e il tempo.

Per altre vesti grafiche ("skins") questi elementi variano in numero e posizione.

Le 16 attività standard di JClic, integrabili solo da persone esperte in programmazione Java, possono essere divise in 9 categorie.

Nei paragrafi che seguono, per ogni attività verrà descritto il settore "*Pannello*" dell'area "*Attività*", l'unico che cambia secondo il tipo di esercizio proposto.

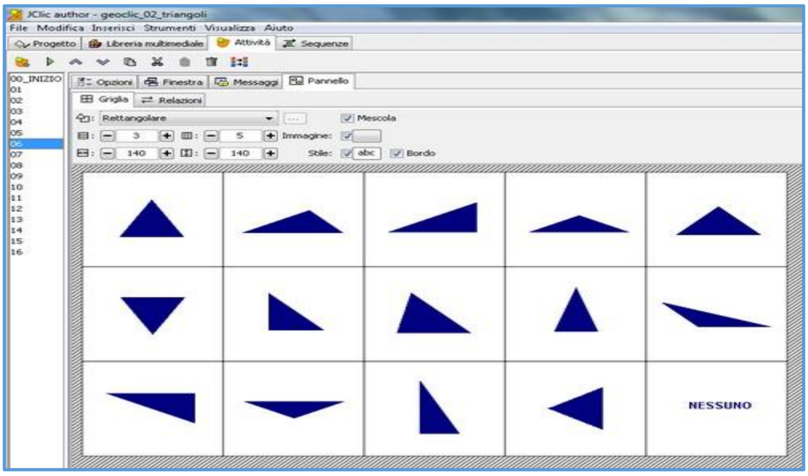

Fig.1 – L’area delle attività

Schermata informativa

La “*Schermata informativa*” è probabilmente l’attività più semplice e ci consente di descrivere accuratamente funzioni di base comuni con molte altre attività e che per questo motivo in seguito non saranno riprese. Come dice il nome stesso, si tratta di un’attività per la fornitura di informazioni all’utente (vedi Fig. 2).

Una tipica schermata informativa è quella posta spesso all’inizio di un progetto con le indicazioni su titolo, autore e finalità degli esercizi. Oppure all’inizio di una sequenza di attività, un testo ricorda, riassume e riporta tutti i dati necessari per la corretta esecuzione degli esercizi che seguono.

Infine possono essere inseriti approfondimenti con testi e immagini al termine di una sequenza.

Si lavora con un pannello di cui è possibile impostare:

1) Righe (es. 3), colonne (es. 2), dimensione orizzontale (es. 300), dimensione verticale (es. 200);
2) Stile (con gli stessi parametri usati per lo stile dei messaggi);
3) Presenza/assenza del bordo;
4) Eventuale immagine di sfondo.

In alto c’è il “*combobox*” per la selezione della forma: la forma rettangolare è la più comune e normale, ma per questa attività è anche possibile usare forme varie tracciate tramite la funzione “*Ritagli*”.

Ogni casella della forma rettangolare o ogni forma ritagliata può essere definita nello stile e nei contenuti, indipendentemente dalle impostazioni generali dell’attività “Schermata informativa”. Così, se abbiamo una griglia rettangolare 3x2, ognuna delle 6 caselle può essere impostata diversamente (ad es. nel colore di sfondo, nell’aspetto del testo, nella posizione delle immagini inserite) semplicemente con un clic sull’elemento interessato.

Quando si operano dei ritagli, si smette di utilizzare una griglia rettangolare e si viene introdotti, cliccando sulla casella con i tre punti vicino a “Ritagli”, in una finestra di lavoro piuttosto completa in cui è possibile:

1) Aggiungere rettangoli, cerchi, poligoni;
2) Selezionare, eliminare, copiare, incollare, ruotare figure;
3) Aggiungere punti su una figura;
4) Unire punti vicini;
5) Fare zoom avanti e indietro per disegnare meglio le forme.

Le forme realizzate sono indicate nella lista sulla destra e possono essere usate come se fossero caselle di una griglia. Si intuiscono le potenzialità di questo strumento, ad es. per la selezione di regioni geografiche o di parti di schede didattiche.

Fig.2 – Schermata informativa

Esplorazione

Nell'attività "*Esplorazione*" se clicchiamo su uno spazio, una casella, un testo, un'immagine, appare un testo o un suono o un'immagine che sono in relazione con l'elemento cliccato (vedi Fig.3).

Rispetto all'attività "Schermata informativa", qui si ha l'introduzione di:

1) Una griglia B accanto alla griglia A (le modalità di modifica sono uguali a quelle già viste per la "Schermata informativa");
2) Una finestra per la disposizione delle griglie A e B (sono previste quattro modalità: AB, BA, A/B, B/A);
3) Una finestra per indicare le relazioni tra le caselle delle due griglie.

Si noti che:

1) Si possono usare griglie rettangolari e forme ritagliate per entrambe le griglie;
2) La funzione "*Mescola*" può essere selezionata sia per la griglia A che per la griglia B;
3) Mentre si inseriscono le relazioni, le frecce possono essere mostrate tutte (in alternativa viene visualizzata solo la freccia dello spazio corrente) e nel colore desiderato.

L'applicazione "*standard*" di questo strumento è la visualizzazione e/o l'ascolto di informazioni relative agli oggetti contenuti nelle varie caselle di una griglia rettangolare. Ma possiamo pensare

anche all'esplorazione delle parti del corpo umano o di un animale o di una pianta oppure alla visualizzazione delle informazioni relative alle varie regioni italiane; in queste attività è evidente come sia preferibile usare forme ritagliate invece di caselle di griglie rettangolari.
La scelta del pannello adeguato, nell'attività di "Esplorazione" come anche in diverse altre attività, dipende dai contenuti e dagli obiettivi dell'esercizio e presuppone una ponderata progettazione iniziale dell'intero progetto Jclic.

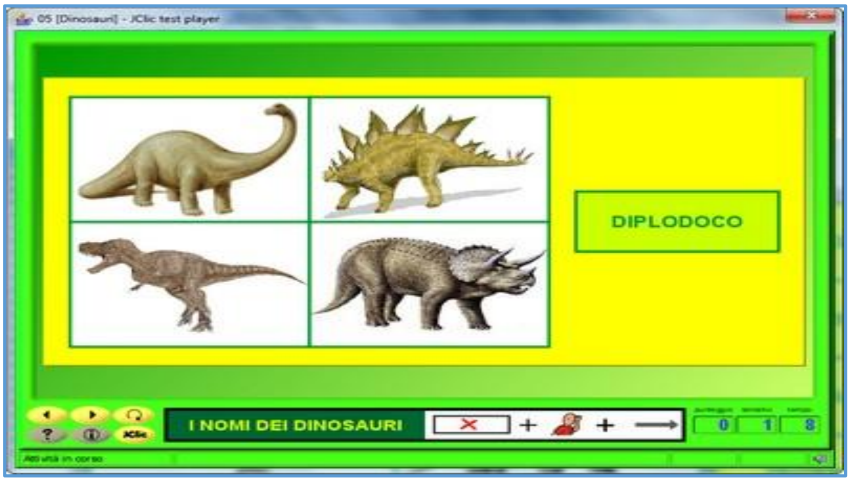

Fig.3 – Esplorazione

Identificare celle

Con "*Identificare celle*" torniamo ad un'attività con una sola griglia, in cui bisogna cliccare sulle forme ritagliate o sulle caselle corrette, definite dall'utente utilizzando la finestra "*Relazioni*" (vedi Fig.4). Semplicemente in quest'ultimo ambiente basta cliccare sulle caselle/forme desiderate ed esse assumono un colore scuro che indica la loro selezione; in fase di esecuzione l'utilizzatore dovrà cliccare solo su quelle celle/forme per completare l'esercizio senza errori. "Identificare celle" ci permette di introdurre un altro elemento interessante, il "*contenuto alternativo*", applicabile alla griglia principale (A) delle varie attività; la funzione si attiva mettendo la spunta in alto a destra vicino alla scritta "Contenuto alternativo:" e consente la definizione di un contenuto diverso sottostante la casella che si identifica correttamente: si può trattare di un messaggio o un'immagine di conferma, di un suono, di una .gif animata... Il contenuto alternativo è particolarmente importante da punto di vista didattico come rinforzo e stimolo per i piccoli utenti e come indicatore "in progress" per la corretta risoluzione degli esercizi.
Si noti che:

1) Per passare dal contenuto "*standard*" delle caselle/forme al contenuto "*alternativo*" si deve cliccare sulla casella "*ALT*" in alto a destra;
2) L'aspetto del contenuto alternativo per le celle di un'intera griglia o delle singole caselle/forme può essere definito all'interno delle corrispondenti finestre "*Stile*".

Come per l'attività di "Esplorazione", anche per "Identificare celle" l'uso di griglie rettangolari o di tante forme ritagliate dipende dall'argomento degli esercizi e dei nostri obiettivi: identificare le parti del corpo di un elefante è un'attività molto diversa dall'identificare i mammiferi tra tante immagini differenti di animali; come anche è molto diverso selezionare tra 10 parole (ognuna in una casella) gli articoli e le preposizioni oppure indicare in un lungo testo (in un'unica cella) avverbi e aggettivi.

Fig.4 – Identificare celle

Scrivi la risposta

"*Scrivi la risposta*" prevede un pannello con due griglie e le funzioni previste sono quelle dell'attività di "Esplorazione" (vedi Fig.5). La differenza principale è che cliccando su una cella della griglia A o su un elemento ritagliato sempre della prima griglia, nella seconda griglia (B) non viene visualizzato il contenuto della casella corrispondente, ma viene richiesta l'immissione di un testo scritto con la tastiera. Un'altra differenza è la possibilità di inserire un contenuto alternativo (nelle modalità illustrate per l'attività di "Identificazione") sempre per le caselle e forme della griglia A. Nella pratica la casella/forma interessata viene evidenziata con un bordo e l'immissione corretta del testo richiesto determina l'annullamento del contenuto principale della cella corrispondente nella griglia A e l'eventuale visualizzazione del suo contenuto alternativo.
Si noti che:

1) Il testo da scrivere va inserito nelle varie caselle facendo attenzione a specificare tutte le varianti opportune (separate dal carattere "|", posto sopra il carattere "\" nella tastiera italiana), in modo da evitare frustrazioni per i piccoli utenti (ad es. per la regione "Valle D'Aosta" si dovrebbero inserire anche le varianti "Valle Di Aosta" e "Val D'Aosta", e per l'alimento "Yogurth" si potrebbero aggiungere pure le forme "Yogurt", "Jogurth" e "Jogurt");
2) È opportuno prevedere per la griglia B una larghezza opportuna dello spazio di inserimento del testo;
3) Di solito le lettere maiuscole e minuscole non influiscono sull'esatta risoluzione degli esercizi;
4) Gli elementi della griglia B possono essere diversi da quelli della griglia A;
5) Un elemento della griglia B può fare riferimento a più caselle/forme della griglia A;

6) Se nella finestra “Relazioni” si seleziona l’opzione “*Risoluzione inversa*”, il gioco termina quando sono stati assegnati tutti gli elementi della griglia B;
7) L’opzione di mescolamento (“*Mescola*”) ha senso (e può essere usata) solo per la griglia A;
8) Bisogna fare attenzione a non inserire spazi alla fine del testo da immettere, mentre i caratteri accentati sono ammessi e riconosciuti;
9) È consigliabile impostare bordi più spessi (quindi aumentare il valore della “*Larghezza del bordo*” nelle varie finestre “*Stile*”) in modo da riconoscere meglio le caselle attive della griglia A.

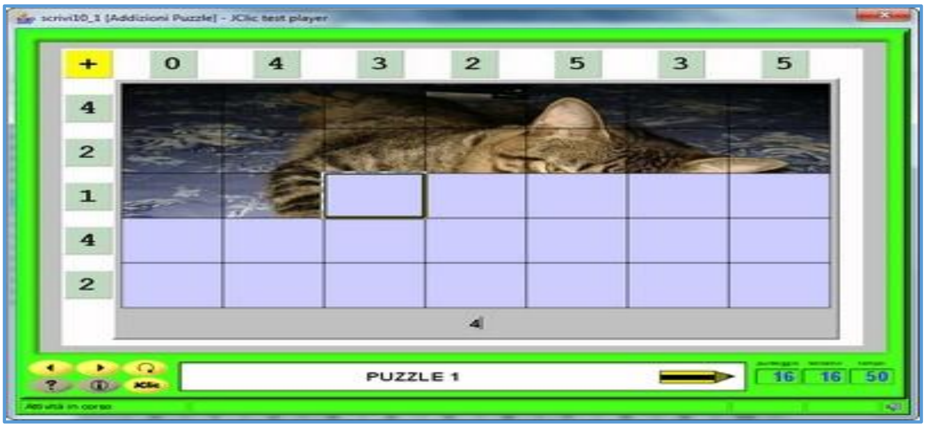

Fig.5 – Scrivi la risposta

Associazione semplice e complessa

In ambito scolastico sono molto utili le due attività relative ai collegamenti presenti in JClic. L’“*Associazione semplice*” permette di associare caselle dello stesso numero: ad ogni casella nella griglia A corrisponde una (e una sola) cella nella griglia B.

L’“*Associazione complessa*” consente anche di associare una cella/forma della griglia B a più caselle/figure ritagliate della griglia A. Nel secondo caso è necessaria la finestra “Relazioni” per l’indicazione dei rapporti tra le caselle delle due griglie (A e B), mentre nel primo caso l’associazione è fissa e si possono decidere solo la disposizione delle due griglie e l’attivazione del mescolamento per esse. Per il resto le due attività presentano le stesse funzioni dell’attività “*Scrivi la risposta*” (vedi Fig.6).

Le applicazioni pratiche delle attività di associazione sono infinite; si possono indicare come esempi un lavoro di collegamento immagini-nomi all’inizio della primaria, un esercizio di collegamento nomi - parti dell’albero, una catalogazione di parole (articoli - nomi - avverbi), tutte attività possibili sia tramite l’“Associazione semplice”, sia per mezzo dell’“Associazione complessa”.

Per l’attività di associazione semplice si noti che:

1) Le due griglie (A e B) sono uguali nel numero di righe e colonne e le modifiche eventualmente apportate in questo senso su una griglia hanno effetto anche sull’altra;
2) Le caselle della griglia A sono associate automaticamente alle caselle della griglia B nelle stesse posizioni.

Per l'attività di associazione complessa è importante sapere che:

1) A una cella/forma della griglia B possono corrispondere più elementi della griglia A, ma non viceversa;
2) Se nella finestra "Relazioni" si seleziona l'opzione "Risoluzione inversa", il gioco termina quando sono stati assegnati tutti gli elementi della griglia B.

Fig.6 – Associazione semplice e complessa

Puzzle doppio, a scambio e a buchi

Tre attività di JClic sono dedicate ai puzzles, in assoluto i più apprezzati dai piccoli utenti delle scuole primarie.

Le verifiche sul campo hanno indicato come sia preferibile utilizzare con frequenza il "*Puzzle doppio*", seguito a discreta distanza dal "*Puzzle a scambio*".

Il "*Puzzle a buchi*" appare invece non molto chiaro nella forma ed è comunque piuttosto difficile da risolvere, soprattutto se le tessere sono numerose (ad esempio 6x5).

In generale i puzzles possono essere usati per produrre esercizi di due tipi:

1) Puzzles classici, per cui un'immagine unica occupa tutta la griglia;
2) Esercizi di riordinamento di qualsiasi tipo (ricostruzione di frasi, ordinamento di numeri e oggetti dal più piccolo al più grande, ecc.), con l'uso di testi, immagini, animazioni e suoni. Ovviamente per il secondo tipo di esercizi il "Puzzle a buchi" è assolutamente da evitare, mentre molto adatto è il "Puzzle doppio".

Per tutti e tre i tipi di puzzle è possibile selezionare (vedi Fig.7), oltre alle griglie rettangolari e alle figure ritagliate, anche:

1) "*Puzzle con unioni a curva*" (tipici pezzi dei puzzles);
2) "*Puzzle con unioni rettangolari*";
3) "*Puzzle con unioni triangolari*".

Si noti che per queste 3 opzioni c'è la possibilità, cliccando sul pulsante con i tre puntini, di attivare/disattivare la distribuzione casuale dei tasselli e di impostare l'altezza e larghezza della loro dentatura.

Il "Puzzle doppio" e il "Puzzle a buchi" prevedono due griglie, il "Puzzle a scambio" solo la griglia A.
Per il "Puzzle a scambio" è consigliabile l'uso di tessere rettangolari, piuttosto che i classici pezzi con indentature.
Infine l'opzione di mescolamento, necessaria nei puzzles, non è disattivabile, mentre è da tenere in considerazione la possibilità di togliere il bordo alle tessere, ad esempio nella ricostruzione di una cartina geografica dell'Italia con le varie regioni.

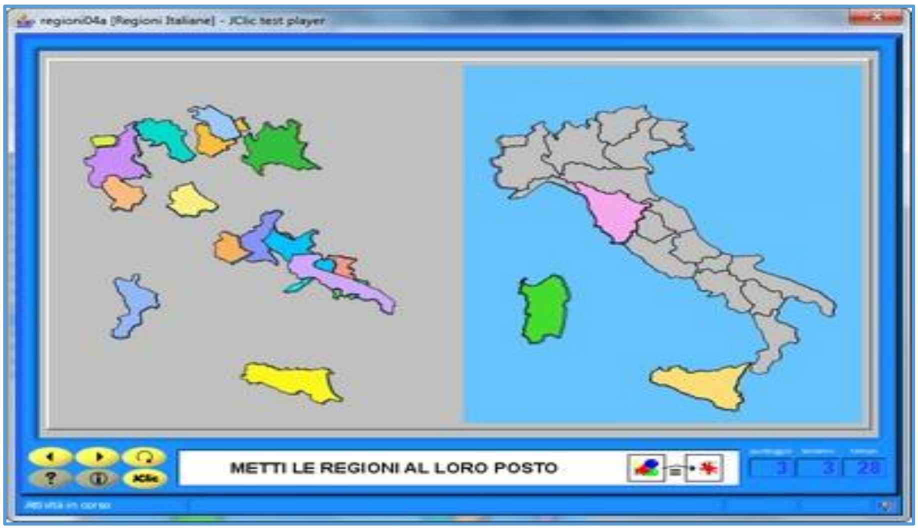

Fig.7 – Puzzle doppio, a scambi, a buchi

Memory

Se le attività di puzzles prevedono l'inserimento di un'unica immagine come sfondo per la griglia A, il memory, per funzionare, deve veder definite singolarmente tutte le caselle.
In ogni cella va messo un contenuto (testuale, grafico, sonoro, misto) che viene automaticamente raddoppiato da JClic (vedi Fig.8).

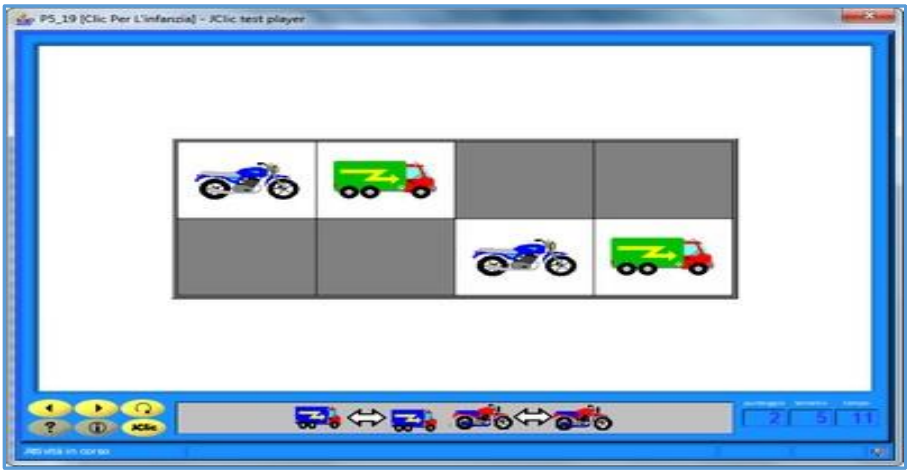

Fig.8 – Memory

Il contenuto alternativo è essenziale per creare esercizi di memory in cui siano associati contenuti di tipo diverso (ad es. il nome di un animale all'immagine che lo rappresenta, ma anche un articolo a un nome).

Crucipuzzle e cruciverba

I due giochi di enigmistica presenti in JClic, "*Crucipuzzle*" e "*Cruciverba*" sono molto interessanti, se non altro per la possibilità di associare immagini alle parole da cercare nel primo caso e alle definizioni nel secondo (vedi Fig.9).

Non esiste un'opzione per la generazione automatica di schemi, ma nel complesso, i due giochi sono più facili da costruire rispetto a Hot Potatoes 6.3, ed è sempre possibile usare programmi appositi per la preparazione di crucipuzzle e cruciverba. In particolare per i cruciverba e i crucipuzzle si possono utilizzare i già descritti Eclipse Crossword, Magnum Opus e The Spellbound! Word Search Creator.

L'attività "Crucipuzzle" si presenta come una griglia 3x3 con le lettere da A a I e con celle larghe e alte 20 pixel. Agendo sul numero di righe e colonne e sulle dimensioni di ogni casella, si può ottenere il diagramma nella dimensione desiderata.

Una volta scritte le parole da cercare nello spazio bianco in alto a destra ("*Parole nascoste*"), esse possono essere aggiunte nello schema.

Dopo il completamento dell'inserimento, in esecuzione JClic provvederà a riempire le caselle vuote con lettere a caso. È comunque consigliabile cercare di completare lo schema per evitare che le lettere inserite a caso vadano a formare parole uguali a quelle già inserite (eventualità possibile, anche se piuttosto remota).

Le parole aggiunte nella lista vengono automaticamente messe in caratteri maiuscoli; e sempre in maiuscolo vengono scritte le lettere nel diagramma.

Si noti che:

1) È possibile eliminare il bordo alle caselle e cambiare l'aspetto di ogni parte del crucipuzzle;
2) Cliccando su "*Usa il pannello B*" si può attivare la griglia B che può essere disposta a sinistra, a destra, sopra e sotto la griglia A e può contenere testi, immagini, suoni e animazioni collegati con la parola corrispondente;
3) Le caselle della griglia B seguono nel loro contenuto l'ordine dei vocaboli nella lista delle parole nascoste;
4) Per avere un crucipuzzle "classico" si può sfruttare un'immagine con parole da cercare da mettere come sfondo alla finestra principale dell'attività; esempi in questo senso si trovano nel pacchetto per la lingua italiana preparato da chi scrive (www.didattica.org/ccount/click.php?id=251);
5) Si può inserire un'immagine unica per tutto il pannello B, in modo che la risoluzione del crucipuzzle porti alla visualizzazione progressiva di un puzzle; la stessa immagine può essere ritagliata (strumento "Ritagli"), in modo che risolvendo il crucipuzzle si possa pervenire al completamento dell'immagine con le forme mancanti (in questo caso conviene togliere il bordo alle forme ritagliate);
6) Le dimensioni dei diagrammi (in righe e colonne) possono essere molto grandi;

7) In fase di preparazione, è ammessa la scrittura del diagramma solo in senso orizzontale.

Nell'attività "Cruciverba" la griglia A iniziale è uguale a quella dell'attività "Crucipuzzle", ma sono presenti subito una griglia B non disattivabile e la finestra della disposizione delle griglie.
Una volta preparato lo schema di un cruciverba agendo opportunamente sulla griglia A (basta scrivere e incrociare le parole, le caselle nere occuperanno gli spazi senza lettere), cliccando sulle caselle iniziali di ogni parola si potranno aggiungere le definizioni sotto qualsiasi forma (testo, immagini, suoni, contenuti misti).
Si noti che:

1) Non ci sono numeri per le definizioni;
2) Possono essere definite anche singole lettere (opzione utile per aiutare gli utenti a risolvere il cruciverba);
3) A differenza dell'attività "Crucipuzzle" non va scritta la lista dei vocaboli;
4) Selezionando "*Separatori trasparenti*" le caselle nere vengono sostituite in esecuzione da spazi trasparenti;
5) Come per l'attività "Crucipuzzle", in fase di preparazione, è ammessa la scrittura dello schema solo in senso orizzontale;
6) Possono essere inseriti cruciverba anche di dimensioni notevoli.

Fig.9 – Crucipuzzle e cruciverba

Attività testuali e linguistiche

Le attività testuali di JClic sono leggermente più complesse, ma permettono comunque di elaborare esercizi efficaci con testi, immagini, suoni e animazioni (vedi Fig.10).
Punti deboli delle quattro attività testuali ("*Completa il testo*", "*Riempi gli spazi*", "*Identifica gli elementi*", "*Ordina gli elementi*") sono:

1) Difficoltà a memorizzare alcune modifiche apportate a esercizi testuali;
2) Impossibilità di disporre correttamente i testi attorno alle immagini (di conseguenza spesso il testo slitta in basso o in alto e le immagini non risultano allineate correttamente alle parole);

3) Necessità di un certo periodo di apprendimento delle procedure di creazione degli esercizi.

La barra dei comandi della finestra “*Stile*” è presente allo stesso modo nei quattro esercizi testuali e contiene le seguenti funzioni:

1) Modifica degli stili del documento;
2) Rimozione degli stili di testo;
3) Impostazione del tipo e della grandezza del carattere;
4) Inserimento di grassetto, corsivo e sottolineato;
5) Centratura del testo o suo allineamento a sinistra o a destra;
6) Colore del testo e dello sfondo.

Similmente sono comuni alle 4 attività testuali quasi tutti i comandi della finestra “*Contenuto*”:

1) Creazione e rimozione di un testo nascosto;
2) Inserimento di una cella con testi e immagini;
3) Impostazione dello schermo precedente e della correzione;
4) Larghezza e altezza in pixel dell’oggetto.

Si noti che:

1) Per l’attività “*Testo - Identifica elementi*”, è possibile scegliere due tipi di attività, “*Identifica le parole*” e “*Identifica i caratteri*”;
2) Per l’attività “*Testo - Ordina elementi*”, è possibile scegliere due tipi di attività, “*Ordina i paragrafi*” e “*Ordina le parole*” (con possibilità di sparpagliare le parole nel paragrafo);
3) Nella finestra “*Schermo precedente*” di default non viene visualizzato alcun testo prima di un’attività, ma si può anche mostrare un testo inserito dall’utilizzatore oppure il testo completo dell’esercizio;

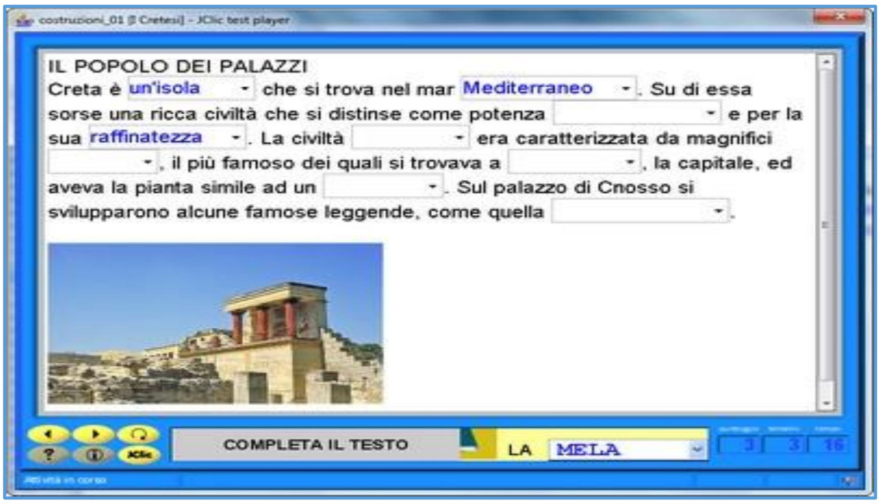

Fig.10 – Attività testuali e linguistiche

4) Sempre nella finestra "*Schermo precedente*" è possibile agire sullo stile e il contenuto del messaggio precedente e sul tempo massimo di visualizzazione dello schermo precedente;
5) Il comando "*Inserisci una cella*" permette l'inserimento di elementi multimediali nell'esercizio e consente la preparazione di attività più stimolanti e interessanti per i piccoli utenti.

Infine, le opzioni di correzione, particolarmente importanti in ambito didattico, riguardano:

1) Per tutte le attività testuali, la presenza del pulsante di correzione (con testo modificabile);
2) Per tutte le attività testuali, tranne "*Testo - Identifica elementi*", l'attivazione o meno di alcuni criteri ("*Considera maiuscole/minuscole*", "*Considera accenti e caratteri speciali*", "*Considera la punteggiatura*", "*Permetti spazi ripetuti*");
3) Per "*Testo - Completa il testo*" e "*Testo - Riempi gli spazi*", l'analisi della risposta;
4) Per il solo "*Testo - Riempi gli spazi*", le possibilità di avanzamento ("*Passa alla nuova parola nascosta una volta completato il riempimento*", "*Passa alla nuova parola solo dopo risposta corretta*").

Uso avanzato di JClic Author

JClic è un programma autore estremamente flessibile e può essere utilizzato per creare facilmente anche attività molto complesse, di solito realizzabili con una certa fatica adoperando i linguaggi di programmazione ad alto livello. In aggiunta è possibile operare variazioni nei file .jclic e .jclic.zip senza avviare JClic Author, in modo da velocizzare l'attuazione di determinate modifiche nei nostri progetti.

Alcune possibili attività complesse creabili con JClic saranno illustrate esaminando vari progetti inseriti nei pacchetti disciplinari presenti nella sezione JClic del sito "*Didattica*" di chi scrive (www.didattica.org/clic.htm).

In generale molte possibilità di creazione di attività complesse sono legate allo sfondo della finestra di esecuzione e soprattutto allo strumento "ritagli" (vedi Fig.11), in combinazione con le attività "Associazione complessa", "Identificazione" e "Puzzle doppio".

Nel pacchetto JClic Immagine (www.didattica.org/ccount/click.php?id=255), descritto nel file www.softwaredidattico.org/files/jclic_immagine.pdf, ci sono:

1) clic_per_l'infanzia (fornitura di informazioni, istruzioni e messaggi senza l'uso di testi);
2) Colori (sfondi vivaci e adeguati e avanzamento tra le pagine anche regolato da timer);
3) I_giochi_dei_colori_1_esploriamo_i_colori (esplorazione di parti di un disegno);
4) memory_animato (memory realizzato con immagini .gif animate);
5) Nonogrammi_5x5_1 (gioco dei nonogrammi preparato usando una griglia A 8x8, con parte attiva 5x5, e una griglia B con i due colori necessari per risolvere il gioco);
6) Quadratini_livello_1 (copia di piccoli disegni fatti con i quadratini).

Nel pacchetto JClic Italiano (www.didattica.org/ccount/click.php?id=251), descritto nel file www.softwaredidattico.org/files/jclic_italiano.pdf, sono presenti:

1) Crucipuzzle_1 (creazione di un crucipuzzle classico sfruttando lo sfondo sottostante il

pannello di gioco per evidenziare le parole trovate);

2) Cruciverba_illustrati_1 (in questi cruciverba da completare con le lettere poste a sinistra e osservando le immagini presenti, sono stati sfruttati uno stile di pagina che dà più spazio agli schemi e lo strumento ritagli per i collegamenti lettere-caselle);
3) riconosci_la_lettera_a (test a 2 e 3 risposte con immagine corretta da cliccare).

Nel pacchetto JClic Lingue Straniere (www.didattica.org/ccount/click.php?id=252), descritto nel file www.softwaredidattico.org/files/jclic_lingue_straniere.pdf, ci sono:

1) Actions_2 (uso massiccio di files audio);
2) christmas_is_here (uso di files audio di diverso tipo, .mp3, .wav e .mid).

Nel pacchetto JClic Matematica (www.didattica.org/ccount/click.php?id=253), descritto nel file www.softwaredidattico.org/files/jclic_matematica.pdf, sono presenti:

1) addizioni_puzzle e sottrazioni_puzzle (riproduzione esatta di due programmi creati da chi scrive in linguaggio Delphi);
2) alberi_di_addizioni (completamento di alberi di addizioni usando il collegamento numeri-caselle);
3) attività_di_calcolo_mentale_livello_1 (applicazione del motore matematica “Arith”);
4) futoshiki_1_4 (griglia A 7x7 per il futoshiki 4x4 e griglia B con i 4 numeri necessari).

Nel pacchetto JClic Musica (www.didattica.org/ccount/click.php?id=249), descritto nel file www.softwaredidattico.org/files/jclic_musica.pdf, si può indicare (le schede didattiche prendono vita) rispondiamo_a_tono_lezione_1.

Nel pacchetto JClic Scienze (www.didattica.org/ccount/click.php?id=254), descritto nel file www.softwaredidattico.org/files/jclic_scienze.pdf, si trova il progetto classe_1_prerequisiti_capacità_percettive_iniziali (tramite lo strumento ritagli vengono animate alcune schede didattiche per la classe prima).

Infine nel pacchetto JClic (www.didattica.org/ccount/click.php?id=250) Storia, Geografia, Studi Sociali, descritto nel file www.softwaredidattico.org/files/jclic_storia_geografia_studi_sociali.pdf, sono presenti:

1) Classe_1_prerequisiti_orientamento_spaziale_1 (schede rese interattive);
2) Regioni_italiane (uso avanzato dello strumento “ritagli” per creare attività interattive riguardanti il nostro paese).

Un’altra possibilità da considerare è quella di modificare direttamente sia il file .jclic sia l’archivio .jclic.zip.

Operare su questi files ci permette in determinate situazioni di risparmiare tempi notevoli.

Ad esempio, se convertiamo tutti i files .gif di un progetto in formato .jpg, non è necessario importarli tutti nella libreria multimediale e quindi cambiare uno per uno ogni collegamento all’interno delle varie attività; sarà molto più veloce:

1) Eliminare tutti i files .gif dall’archivio .jclic.zip e sostituirli con i corrispondenti files .jpg;
2) Estrarre dallo stesso archivio il file .jclic e modificarlo in un editor ASCII, sostituendo tutte le stringhe di testo “.gif” con la stringa “.jpg”;
3) Salvare il file .jclic modificato e sostituire il file originale presente nell’archivio

.jclic.zip.

Fig.11 – Un uso dello strumenti "Ritagli"

Naturalmente:

1) Queste operazioni possono essere effettuate se il file .jclic.zip non è aperto correntemente da JClic Author;
2) Prima di modificare .jclic.zip crearne una copia di salvataggio. Si noti che il file .jclic è in formato "aperto" XML e ogni sua parte può essere modificata.

Novità nella versione 0.3.1.0

JClic (http://clic.xtec.cat/en/jclic/download.htm) nella nuova versione permette l'esportazione dei progetti direttamente in HTML5 per la loro esecuzione in rete tramite i navigatori Internet e senza bisogno di Java o Flash installato.

È una possibilità eccezionale per la pubblicazione on-line dei nostri progetti.

Centinaia di esempi si trovano in http://edilim1.altervista.org/index.html oppure http://jclic1.altervista.org/index.html; bisogna però tenere presente che il player web è molto semplice e ci sono alcune limitazioni che indico di seguito (e verificate durante le mie operazioni di conversione):

1) Il progetto (del lettore) è un lavoro in corso: solo alcune funzioni base di JClic sono implementate;
2) C'è solo un'interfaccia di base, senza contatori o pulsanti (ci sono solo le frecce per andare avanti e indietro); in particolare sono assenti i pulsanti "Aiuto" e "Ripeti l'attività";
3) Le funzioni di report non sono ancora implementate;
4) I suoni degli eventi non sono ancora implementati (funzionano invece i suoni all'interno delle attività);
5) Non bisogna usare files .wav, ma solo files .mp3;
6) La riproduzione dei video e la registrazione dei suoni non sono ancora implementate;
7) Le attività testuali "Completa il testo", "Ordina gli elementi" e "Identifica gli elementi"

non sono ancora implementate;

8) Nell'unica attività testuale funzionante, "Riempi gli spazi", la risposta corretta non deve essere messa al primo posto se si usa il menu a tendina e non bisogna attivare mai il tasto "*Verifica*";
9) La riproduzione dei files MIDI non è ancora implementata;
10) Conviene non convertire progetti più grandi di 10 Mb;
11) Il lettore on-line è testato solo con le ultime versioni di Chrome/Chromium e Firefox;
12) Le attività con ritagli complessi devono essere verificate caso per caso;
13) Conviene usare caratteri abbastanza grandi in tutte le attività (16-22 in genere per tutte le attività, almeno 20 per le attività "Scrivi il testo", "Crucipuzzle", "Cruciverba" (schema e definizioni);
14) Facciamo in modo (ma questo vale anche per i progetti non on-line) che le dimensioni totali delle finestre delle varie attività sia 800-1000 in orizzontale e 600-700 in verticale.

Praticamente se vogliamo convertire i progetti JClic per il web, dobbiamo scordarci dei suoni degli eventi, dei file MIDI e WAV, dei video, dei report, dei pulsanti sull'interfaccia (escluso avanti e indietro) e delle attività di testo "Completa", "Ordina" e "Identifica"; dobbiamo poi usare caratteri piuttosto grandi e verificare caso per caso le attività con i "ritagli".
Al di là di questi limiti, che saranno sicuramente corretti in versioni successive del programma, è possibile verificare in http://jclic1.altervista.org/index.html la qualità dei prodotti pubblicabili.

Conclusione

Per approfondire l'uso del programma si può fare riferimento ai seguenti link:

1) http://www.didattica.org/clic.htm;
2) https://www.facebook.com/groups/752682221508500/?ref=bookmarks (vedi Fig.12);
3) http://www.ivana.it/formazioni/Jclic/;
4) http://jclic1.altervista.org/index.html.

Fig.12 – Il gruppo Facebook "Progetti JClic in italiano"

Dichiarazione di conflitti di interesse

Gli autori dichiarano di non avere conflitti di interesse rispetto la paternità o la pubblicazione di questo articolo.

Deposito dei materiali dell'attività

Al seguente link sono depositati eventuali materiali inerenti questo l'articolo. Questi materiali nel tempo potranno essere modificati e arricchiti seguendo l'evoluzione delle idee sottostanti o/e future sperimentazioni svolte dall'autore dell'articolo.

http://www.edimast.it/J/20160201/02690284MU/

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Contents

Experiens & Research Articles

Book & Software Reviews