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article

A Journey through the geometrical alchemies of art: an example of constructivist teaching

Annarosa Serpe, Maria Giovanna Frassia

Abstract. *This paper illustrates a teaching setting which adopts a constructivist approach, and is centered on the combination of Mathematics and Art. The aim is to appeal to those minds that have a curiosity for a discipline which has long interacted with artistic expression, and to introduce new perspectives in the teaching of Mathematics in the light of the latest technological challenges. The trigger element - a decorative spiral frieze - has led to a process of Euclidean geometrical construction first with traditional tools, and then through the use of simple algorithms implemented in the MatCos programming environment. The drawing of the spiral curve represents a learning opportunity, a practice which combines analysis and synthesis, a journey between past and present which offers the student the opportunity to acquire some basic conceptual and instrumental implications in the age-old relationship between Mathematics and Art.*

Key words. *Archimedes' Spiral, Architectural Frieze, Constructivist Teaching Approach, Programming Environment.*

Sommario. *L'articolo propone una situazione didattica di taglio costruttivista imperniata sul binomio Matematica-Arte allo scopo di avvicinare le menti curiose a quella matematica che ha profondamente interagito con l'espressione artistica e, in virtù dalle nuove istanze poste dalle tecnologie, di introdurre nuove prospettive nell'insegnamento della Matematica. L'elemento attivatore, identificato nei fregi decorativi a forma di spirale, consente di mettere in atto un processo di costruzione geometrica euclidea prima con strumenti tradizionali, poi mediante semplici algoritmi implementati nell'ambiente di programmazione MatCos.*
Il disegno della curva spirale costituisce un'occasione conoscitiva, una pratica che racchiude in sé analisi e sintesi; un cammino tra passato e presente che offre allo studente l'opportunità di cogliere alcuni lineamenti concettuali e strumentali dell'atavico rapporto tra Matematica e Arte.

Parole chiave. *Ambiente di programmazione, Approccio didattico costruttivista, Fregio architettonico, Spirale di Archimede.*

Introduction

Today more than ever the teaching of Mathematics is motivated by the contribution it can give to the shaping of thought and personality as a whole. Teaching means taking into consideration not only the cultural role which pertains to Mathematics, but also the motivation which sustains this role in an ever changing society. Consequently, didactics needs to be practical and aim to recompose analytical knowledge, to re-affirm that all knowledge is part of a whole. Only in its entirety can it improve the understanding of reality. This calls for a re-visiting of traditional teaching, which is based on the transmission of notions, and which views learning as receptive elaboration of data, independent of others, and therefore solitary.

Constructing informed knowledge instead implies integrating different areas of interest which interact like mental models derived from cognitive science, co-operative learning and technological fields. The teaching of mathematics cannot neglect the synergy with other subjects. Research studies have indeed shown how the integration and harmonization of contents pertaining to different disciplines represent viable paths towards informed and lasting learning (Adelman, 1989; Check, 1992). Interdisciplinary connections in the educational process facilitate a more thorough comprehension of concepts, because the understanding can get to the root of notions and avoid 'airtight compartments' (Brooks & Brooks, 1993).

Interdisciplinary learning breaks the linear model of knowledge development and opens the door to constructivist didactics in which subjects display their full educational value within learning environments derived from the real world and based on cases, rather than on pre-determined instruction sequences (Wicklein & Schell, 1995). A constructivist type of didactics emphasizes the construction of knowledge and not its repetition, and facilitates reflection as well as reasoning, also through collaboration among peers (Jonassen et al., 1998).

Learning environments with the above-mentioned aims allow the students to experience a process of flexible re-adjustment of pre-existing knowledge according to the needs of the new educational experience.

In this paper, the authors put forward an example of an inter-disciplinary learning environment centred on the Mathematics-Art combination. A similar proposal might appeal to those minds that have a curiosity for a discipline which has long interacted with artistic expression, and could open up new perspectives in the teaching of Mathematics in the light of the latest technological challenges.

Theoretical Framework

In Italian higher secondary schools the connection of Mathematics with real life is an often stated truth. However, this connection is considered too difficult for the students to experience, except in a few standard cases mentioned in most textbooks. Some topics, for example, are avoided because they are difficult to manage during classes, especially as far as their visualization and graphic representation are concerned.

One example is given by algebraic curves. If we exclude the classic teaching paths on conics, other algebraic curves such as spirals, epicycloids, hypocycloids, etc., are usually ignored. This represents a gap in the learners' education because curves - geometrical objects *par excellence* - in the mathematical imagination play the delicate role of a border zone where different and sometimes opposed activities converge.

Curves offer a rich and partly unexplored field of study that can be discovered and rediscovered also at inter-disciplinary level, because they combine tangible objects and practical mechanisms with abstract mathematical thought; aptly included in classroom teaching, they help students to identify correlations between geometrical concepts, the mechanisms of technology and science constructions. As curves refer back to drawing, the latter becomes very important, located as it is between concrete reality and the abstract geometrical concept it represents; as a result, drawing is the basis of the dialectic process and relationship between the figural and the conceptual components (Mariotti, 1995).

A drawing represents an object observed and summarized in a few traits. It is a learning opportunity, a practice which combines analysis and synthesis. Indeed the tracing of a curve, defined as a locus of points which satisfy a given propriety, usually passes through the geometric construction of one of its points with classic tools like ruler and compasses.

However, today such tools can be integrated or substituted with recent educational software which allows the execution and manipulation of geometric constructions (Serpe, Frassia, 2015).

Such software programmes turn the classic traditional principles into information technology. Their use in the teaching - learning of Mathematics has been shown in the literature (Doerr, Zangor, 2000; Ruthven, Hennessy, 2002; Artigue et al., 2003; Lagrange et al., 2003; Monaghan, 2004; Lagrange, Ozdemir Erdogan, 2009; Dix et al., 2004; Guzman, Nussbaum, 2009; Collins et al., 1988; Costabile, Serpe, 2012 and many other).

The representation of a curve can also be seen as a visually beautiful shape, thereby connecting Mathematics and artistic expression. Mathematics and Art, expressive linguistic modes which are quite distant, are - despite appearances - connected to each other, because both subjects involve the imagination and creativity.

The present work fits into this theoretical framework, and puts forward a constructivist teaching practice. Here visual art and geometry meet in a technological setting through the practice of programming, whose pedagogic and formative value is known in the literature (Pea, Kurland, 1984; Kurland et al., 1989; Oprea, 1988; Serpe, 2006 and many other).

The trigger element was identified as a decorative spiral frieze, which led to a process of Euclidean geometrical construction first with traditional tools, and after through simple algorithms implemented in the MatCos[1] programming environment.

A similar path uses design as a tool for formal analysis between past and present and joins a tangible object to abstract mathematic thought. It is a learning path in which students realize how mathematical reasoning and art share analogies, how shapes derived from empirical solutions can stand tests with deeper meanings, reasons and evaluations.

Methodology and Design Activity

The learning environment was set up with the specific aim of guiding students to the understanding of the relationship between concrete versus abstract, staying away from purely theoretical mathematical concepts. The activity begins with the analysis of the decorative frieze on the façade of the *San Matteo al Cassaro* church in Palermo (Italy), and later moves on to the geometric investigation which has the construction of concepts and meanings as its objective.

The design implies the study of the geometrical method for tracing Archimedes' spiral in Euclidean terms using drawing tools like ruler, set square and compasses and later also with a computer as a programming tool. We chose to use computer programming because it is a constructive and creative activity, which reinforces the acquisition of concepts as well as abstraction skills.

The construction of the Euclidean curve represents a strategic phase from the teaching point of view because it helps the learner to perceive the complexity of the whole, which starts from 'simple and evident' and arrives at the 'complex and not evident'. Geometric construction indeed helps students to start this process which will take them from easy to difficult in a tangible, critical and rigorous way.

The re-interpretation of data through information technology tools enables the learner to turn the meaning of a representation into its construction (filling the gap between experience and creation) through a model which shows and communicates geometrical synthesis.

Mathematical modelling has expressive and evocative potential from the metric and formal point of view; the virtual tracing of a curve through simple algorithms adds value to the experience:

«*The interaction between a learner and a computer is based on a symbolic interpretation and computation of the learner input, and the feedback of the environment is provided in the proper register allowing its reading as a mathematical phenomenon*». (Balacheff & Kaput, 1996, p.470).

Such methodology avoids the traditional pitfalls of Mathematics classes, and gives students a true chance to stretch their views.

The geometric reading of a frieze: Archimedes' spiral

Identifying the trigger element: the decorative spiral frieze

The teaching sequence begins with a guided virtual tour of the city of Palermo where the students are attend one of the first two years of higher secondary school.

With the help of their Drawing and Art History teacher, they discover the city's artistic and architectural heritage. During the virtual tour the students are asked to pay special attention to the decorative elements on the façades of buildings. This way the students have the opportunity to observe and recognize some special geometrical shapes in the architectural friezes. During the observation phase an extraordinarily beautiful shape - which develops with an enveloping movement - is identified in the large coils found on the façade of the *San Matteo al Cassaro* church[2] (see Figure 1).

Fig.1 - The façade of the *San Matteo a Cassaro* church

First the Drawing and Art History teacher sets up a discussion which highlights how this enveloping shape - called spiral - is a figure created on the basis of a more or less complex plan to transfer information about real or imaginary objects, concepts and emotions; so the matter is worth investigating.

Here the Mathematics teacher steps in, in order to take the work further, as the students progress from their experience of discovery to research pathways where they find out more, and learn about, the architectural frieze (see Figure 2).

Fig.2 – Detail of a large spiral coil

Developing the empirical situation: geometrical reading of the frieze

Developing the empirical situation through a process of mathematical enables the learners to make a considerable leap in abstraction: the concise and objective description of the curve requires the introduction of concepts and tools that are acquired and tested in the model study phase. Later the evaluation of the model allows to perfect the spiral construction algorithm, as well as a reflection on the theoretical aspect and further needs analysis. For all the above reasons, the mathematical modelling process consists of three steps.

In the first step the students draw a curve using the traditional tools, ruler and compasses, and later proceed to use the virtual tools through the MatCos[3] The second step is very relevant from the point of view of abstraction because it requires a refining of the previous construction process through the construction of the ‘for’ cycle or iteration.

The last step has the aim of further perfecting the construction of the curve. Such a highly educational process trains the students in the appreciation of the potential of the language of Mathematics and at the same time offers them a reading awareness of theories.

The steps are described in detail below.

<u>Step 1</u>

The class is divided in groups and each group receives the tools and material for the task (sheet of paper, ruler, compasses, pencils and rubbers); afterwards, the teacher sets the following task:

1. Construct the square *ABCD*;
2. Determine the median points (*E*, *F*, *G*, *H*) of the sides of the square;
3. Construct the rays (*e, h, g, f*) with origin the median points of the square and passing by one of the vertex of the square and belonging to the side on which the median point itself lies (*anticlockwise direction*);
4. Construct the circumference with centre *A* and passing by point *B*;

5. Determine the point of intersection *I* between the circumference and the ray with origin the median point of side *AB* and passing by point *A*;
6. Draw the arc *p* with centre *A* and points *B* and *I* as extremes;
7. Repeat from 5 a 7:
   I. Construct the circumference *CC* with centre *A, B, C, D* (in this order) and radius the distance between the vertex of the square and the point of intersection between the old circumference and the rays *e, h, g, f* (in this order);
   II. Construct the arc with centre in the vertex of the square.

The above-mentioned geometric construction is executed four times in order to complete the first turn of the spiral; after that the students should implement it in the MatCos[4] programming environment (see Fig.3).

/*Programme Code MCS1*/

```
A=point; B=point; l=segment(A,B);
p1=perpendicular(l,A); D=EndPoint(p1,/2); p2=perpendicular(l,B);
C=EndPoint(p2,2); s=segment(C,D);
E=Midpoint(l); F=Midpoint(p1); G=Midpoint(s);H=Midpoint(p2);
e=ray(E,A); f=ray(F,D); g=ray(G,C); h=ray(H,B);
PenColor(0,128,0);PenWidth(5);
r1=Distance(A,B); cc=circ(A,r1);
I1=intersection(cc,e); p1=arc(A,B,I1,ANTICLOCKWISE);
r2=distance(B,I1); cc=circ(B,r2);
I2=Intersection(cc,h); p2=arc(B,I1,I2,ANTICLOCKWISE);
delete(cc);
r3=distance(C,I2); cc=circ(C,r3);
I3=intersection(cc,g); p3=arc(C,I2,I3,ANTICLOCKWISE);
delete(cc);
r4=distance(D,I3); cc=circ(D,r4);
I4=intersection(cc,f); p4=arc(D,I3,I4,ANTICLOCKWISE);
delete(cc);
```

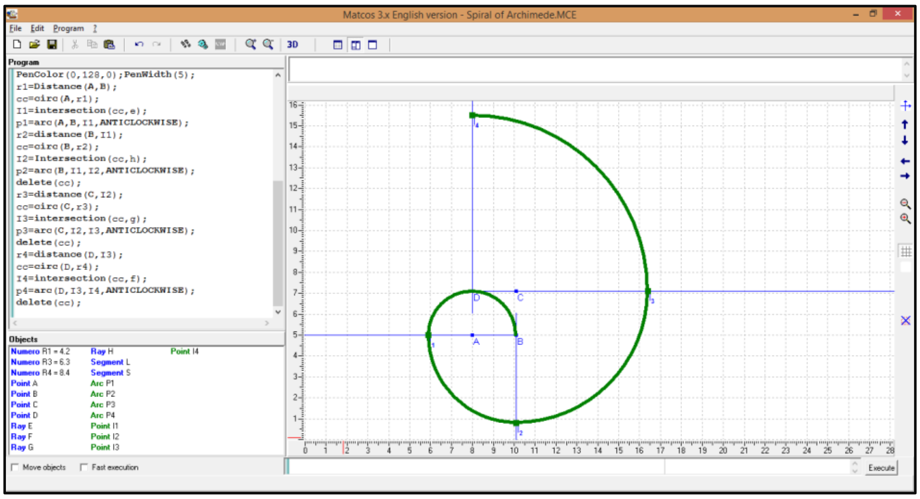

Fig.3-Output of the first coil

Materials, tools and task get the students going until they achieve the geometrical reading of the decorative frieze. The change from traditional to virtual instruments adds value to the learning path: the MatCos programming environment is used just like a real drawing pad, but it allows a more accurate syntax of the Euclidean register.

Step 2

The construction algorithm enables the students to better learn the characteristics of the curve. However, its repetitive execution could be boring if one wanted to reproduce the decorative frieze fairly accurately.

From this derives the need to shorten the sequence of repetition (the drawing of the joined arcs). The previous construction algorithm can be refined making use of the 'for' option to draw the joined arcs. The new formalized algorithm (see Fig.4) is:

1. Draw two points *A* and *B*;
2. Construct the square with side *A* and *B*;
3. Determine the median points of the sides of the square;
4. Construct the rays with origin the median points of the square and passing by one of the vertexes of the square and pertaining to the side on which the median point lies (*anticlockwise direction*);
5. Construct the circumference with centre *A* and passing by point *B;*
6. Determine the point of intersection *I* between the circumference and the ray with origin the median point of side *AB* and passing by *A*;
7. Trace the arc *p* with centre *A* and extremes the points *B* and *I*;
8. Cycle for the construction of the joined arcs.

/*Programme Code MCS2*/

```
n=readnumber;
A=point; B=point; l=segment(A,B);
p1=perpendicular(l,A); D=Endpoint(p1,2);
p2=perpendicular(l,B); C=Endpoint(p2,2);
s=segment(C,D); polygon(A,B,C,D);
E=Midpoint(l); F=Midpoint(p1); G=Midpoint(s); H=Midpoint(p2);
e=ray(E,A); f=ray(F,D); g=ray(G,C); h=ray(H,B);
PenColor(0,128,0);PenWidth(5);
r=distance(A,B); cc=circ(A,r);
I1=intersection(cc,e); arc(A,B,I1,ANTICLOCKWISE);
delete(cc);
for(i from 1 to n) do;
P=SelectObject; Q=SelectVertex;r=distance(P,Q);
semirett=SelectObject; cc=circ(Q,r);
I1=intersection(cc,semirett); arc(Q,P,I1,ANTICLOCKWISE);
delete(cc);
end;
delete(semirett,I1,P,Q);
```

Refining the previous process of construction is relevant because the students are required to make a considerable abstractive leap: what is difficult is the identification of the construction, within the algorithm, which enables the repetition of an operation (construction of joined arcs) as long as a certain condition is true.

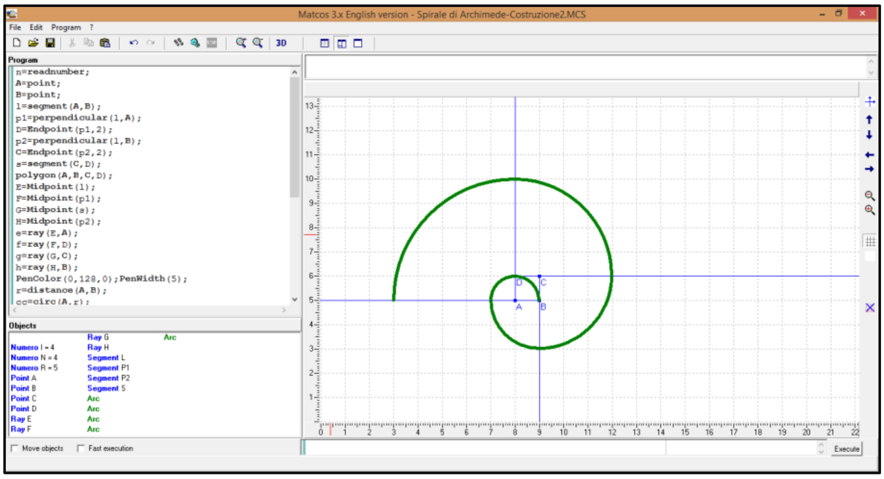

Fig.4-Output of the refined algorithm

<u>Step 3</u>

At the end of the second phase it is important to reflect on possible further improvements; once again the students are encouraged to make another abstraction leap. Careful observation leads the participants to notice that the second extreme of each joined arc can be obtained from the rotation of the first extreme by 90°; that is why the construction of the rays starting from the vertices of the square, of the circumference and of the intersection point between the half-line and the circumference becomes superfluous. Consequently, the final formalized algorithm (see Figure 5) is as follows:

1. Draw two points *A* and *B*;
2. Draw point *A'*, obtained from the rotation of *A* with respect to centre *B* by a 90° angle anticlockwise;
3. Construction of the arc with centre *B* and extremes *A* and *A'*;
4. Cycle for the construction of the joined spiral arcs with appropriate choise of the required objects at point 2.
5. /* Programme Code MCS3*/

```
n=readnumber;
A=point; B=point; l=segment(A,B);
p1=perpendicular(l,A); D=Endpoint(p1,2);
p2=perpendicular(l,B); C=Endpoint(p2,2);
s=segment(C,D); polygon(A,B,C,D);
PenColor(0,128,0);PenWidth(5);
P=SelectObject; Q=SelectVertex;
PP=rotate(P,Q,90,ANTICLOCKWISE); arc(Q,P,PP,ANTICLOCKWISE);
for(i from 1 to n) do;
P=SelectObject; Q=SelectVertex;
PP=rotate(P,Q,90,ANTICLOCKWISE);
arc(Q,P,PP,ANTICLOCKWISE);
end;
delete(A,B,C,D,P,PP,Q);
```

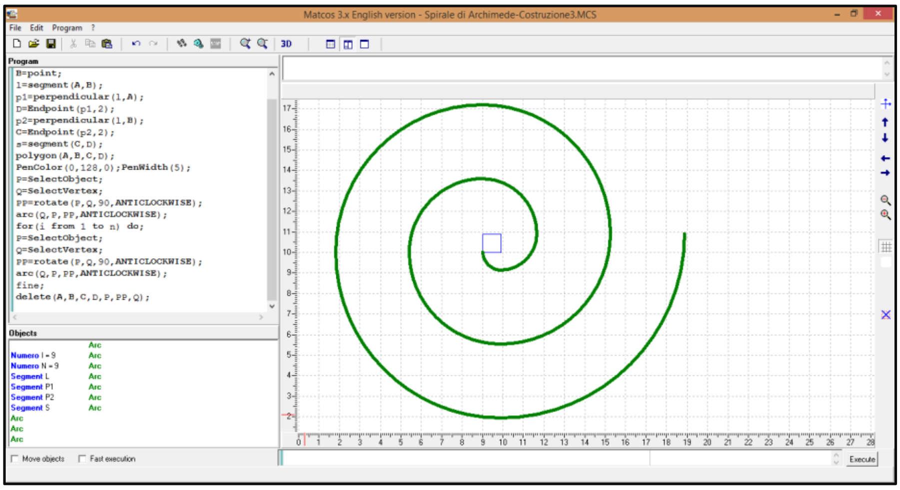

Fig.5 - Final Output of Archimedes' spiral

Discussion

The topic unfolds around the enveloping movement of the spiral geometrical shape which represents a recurring theme in Art, and in the synchronic interpretation of the geometrical connections to be identified.

The constructivist approach allows the students to work on evolving shapes. In the first step we noticed that the students showed different degrees of manual skills when drawing with traditional tools like ruler and compasses. This is almost certainly due to the process of idealization where material instruments become abstract objects. That is why we got the learners to spend quite some time on the virtual manipulation of geometrical shapes to increase their awareness of them. The implementation of the first construction algorithm for the curve, repeated several times, facilitated the understanding and use of the geometrical objects; however, at the same time the need to shorten the repetition sequence for the construction of the joined arcs emerged for the first time. At this point along the activity interaction among the students in their groups had considerably improved. They developed a spirit of co-operation and solidarity among peers far greater than previously.

This is the result of group work, where everyone feels the need to be accepted so everyone gets involved. Only two groups worked on the improvement of the algorithm, and they immediately shared their findings with the rest of the class without any intervention by the teacher. This 'coaching between peers' is a valid learning strategy because skills and knowledge are acquired through direct personal effort and emotional involvement.

Conclusion

One of the main difficulties in teaching and communicating Mathematics is its abstract nature. In order to make it concrete and more easily understandable to students it is essential to use different strategies that can reveal the crucial role Mathematical reasoning plays in our lives, and may uncover the beauty of the subject. The setting up of a similar learning environment is an attempt in this direction, and aims to show how artistic expression and mathematics, apparently pertaining to very distant spheres of knowledge, can indeed be connected because they both require imagination and creativity. Combining these two expressive forms into a teaching situation has the effect of raising interest for Mathematics and awakening mathematical aptitude.

It is a motivating initiative, which provides stimulation and offers the tools for different learning styles. It also encourages self-learning through creativity and peer-teaching and - starting from previous notions - helps the students to interpret their own experiences and construct more complex and formally articulated knowledge and competences.

To conclude, this constructivist teaching initiative is to be intended as a fascinating journey through Art's geometrical alchemies which will break down the cultural barriers that have long existed between the two disciplines in school education, while at the same time recovering classical notions through the use of methods which are up-to-date and which can captivate the students' attention.

Declaration of Conflicting Interests

The authors declared that they had no conflicts of interest with respect to their authorship or the publication of this article.

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http://www.edimast.it/J/20160201/02490260SE/

Note

1. The MatCos programming environment, now in its 3rd version, was devised and created on the following pedagogic-educational paradigm:
 - Introducing the learners to computer programming using mathematical concepts suitable for their age group;
 - helping students to learn mathematical concepts and methods exploiting the potential of computers.

For specific educational aspects and technical requirements, you can visit the Web Site Url: http://cird.unical.it/.

2. The Church of *San Matteo al Cassaro* is a place of worship in the historic centre of Palermo that gives out onto the street *del Cassaro* (now Vittorio Emanuele Avenue). In the three floors of the principal façade, completed in 1662 and richly decorated, three niches were carved out which house the statues of *La Vergine* in the middle, *San Matteo Apostolo ed Evangelista* on the left and *San Mattia Apostolo* on the right. Above the portal, on the second floor an elaborate cornice separates the two levels and on the external sides features large, spiral, connecting coils which can be found also on the third floor. The church was built by the architects and sculptors Carlo D'Aprile, and Gaspare Guercio who followed in Vincenzo Guercio's family footsteps.
3. The students know the MatCos programming environment quite well because it is used for the Mathematics curriculum activities.

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About the Authors

Annarosa Serpe
Department of Mathematics and Computer Science
Via Ponte P. Bucci, CUBE 30/A, 87036 Rende (CS)
annarosa.serpe@unical.it
Italy

Assistant Professor in the Department of Mathematics and Computer Science of the University of Calabria (UNICAL). She carries out scientific research in the teaching-learning of mathematics with technology, with particular reference to the use of the computer as a tool for programming and computational transposition of mathematics and related epistemology, the design and implementation of computer-based interactive learning environment. She is the author of several publications and co-author of two multimedia software packages for teaching and learning in primary schools. For more details see the web site url: https://www.mat.unical.it/~serpe/

Maria Giovanna Frassia
Department of Mathematics and Computer Science
Via Ponte P. Bucci, CUBE 30/A, 87036 Rende (CS)
frassia@mat.unical.it
Italy

PhD student in Mathematics Education. Her interest in research started after her degree in Mathematics, when she attended seminars about didactics of Mathematics with technology, in the Interdepartmental Center of Didactic Researches (C.I.R.D.) of University of Calabria, She later obtained two Masters degrees and teaching qualification for class A049 (Mathematics and Physic in high school). Her research interests include the use and the role of technology in classroom practice with particular reference to software-oriented Mathematics education.

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