

# Intuitive Geometry by Emma Castelnuovo: still contemporary in the digital devices' era

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**Abstract** Emma Castelnuovo has been one of the most influent innovators of the Italian mathematics Education. She published for the first time her book “Intuitive Geometry” in 1949, by which she presented a completely new perspective of geometry teaching/learning. Castelnuovo’s work had a strong impact on mathematics education. Nowadays, educational innovation is often confused with the use of digital devices; so it would seem that intuitive geometry by Castelnuovo, based on material models manipulation, is destined to have no more room in our classes. In this paper, we support the opposite conclusion: dynamic geometry softwares allow performing those explorations, which have always been promoted by Castelnuovo but with some differences. A classroom activity in grade 6 is used as example to support our conclusion.

**Key words** Castelnuovo, DGS, GeoGebra, Intuitive geometry.

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**Sommario** Emma Castelnuovo è stata forse una delle più influenti innovatrici della didattica della matematica in Italia. Nel 1949 pubblicò, per la prima volta, la sua opera “Geometria Intuitiva” in cui presentava l’insegnamento/apprendimento della geometria euclidea con un volto completamente nuovo rispetto a quello tradizionale. L’opera della Castelnuovo ha avuto un impatto fortissimo sulla didattica della matematica. Attualmente l’innovazione didattica viene spesso confusa con l’uso di strumenti digitali; pertanto potrebbe sembrare che la geometria intuitiva della Castelnuovo, basata sulla manipolazione di oggetti concreti, sia destinata a non aver più spazio nelle nostre classi. Nell’articolo si difende invece la tesi opposta: i software di geometria dinamica permettono molte di quelle esplorazioni che la Castelnuovo ha sempre promosso, ma con alcune differenze. Un’attività svolta in una classe prima di scuola secondaria di primo grado servirà da esempio a supporto della tesi.

**Parole chiave** Castelnuovo, GeoGebra, Geometria intuitiva, Software di geometria dinamica.

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## Intuitive geometry by Emma Castelnuovo

Emma Castelnuovo (1913-2014) was a teacher of mathematics in middle school. She was born in Rome, her father was Guido Castelnuovo and his uncle was Federico Enriques; both of them were very well known mathematicians and spent their energies for the development of education in mathematics. Due to the Italian racial laws, she could start her activity as a teacher only after the Second World War and she soon started to dedicate her effort for the improvement of mathematics teaching at the time. One of her most famous publication is “Intuitive Geometry” (Castelnuovo, 1949-1964) a textbook in which her methods for geometry teaching and learning are articulated in many different tasks, constructions, exercises. She defines her method as *continuous*, because it is based on students’ previous knowledge, and *active*, because of the usage of experiments and the involvement of students’ discoveries (Furinghetti, Menghini, 2014).

Intuitive Geometry, as it is described by Castelnuovo, is characterized by the high connection with reality, the frequent use of manipulatives of different kinds (as folding cards, strips, cords ...) with the aim of leading students to their own discoveries of the geometrical shapes’ properties. Her view of intuitive geometry is clarified from the very first lines of the introduction, in which she stresses that

“ [...] We begin from material experiences and from the construction of models with the aim to elicit the figure’s image; but the child’s attention will not be focused on the model but on the processes which conducts to a particular construction and on those which can be performed changing some elements of the already made model. Such experiences and processes will made the children more and more aware of the

limit of the concreteness. In such a way he will be led, almost by himself, to detach from the tangible constructions to arrive, through the consideration of continuously variable figures and in particular of the “limit” cases, to the generalization of a property and so to abstraction. (Castelnuovo, 1964, p.III)

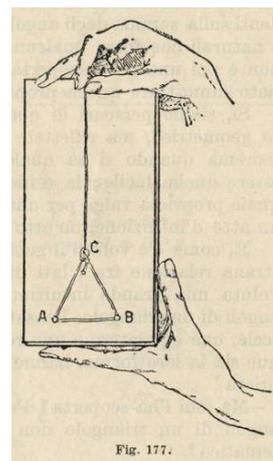
It is interesting to notice that this idea of intuitive geometry is never contrasted to deductive geometry. In the book it is specified that the use of models has the aim of letting the pupils experience the intuition, which usually precedes the formal proof and that creates the need of the proof itself.

To better understand what is meant by the former words, an extract from the book is here transcribed:

Su una tavoletta, di dimensioni scelte a piacere (fig. 177), siano piantati due chiodi A e B. Attorno ai chiodi passa un elastichino, di cui un ramo viene tirato, con uno spago fissato nel punto di mezzo, in direzione perpendicolare alla congiungente i chiodi (che è realizzata dall'altro ramo dell'elastico).

Si ottengono così tanti triangoli isosceli di base fissa AB e vertice C variabile. Questi triangoli hanno tutti la stessa base mentre l'altezza relativa alla base varia al variare della forza con cui si tira l'elastichino. È interessante studiare gli angoli di questi triangoli al variare dell'altezza. Se si immagina di partire dal triangolo più grande che si può realizzare sulla tavoletta e di allentare a poco a poco lo spago, l'angolo al vertice  $\hat{C}$  diventa sempre più grande mentre gli angoli alla base  $\hat{A}$  e  $\hat{B}$  diventano sempre più piccoli: se dunque due angoli di un triangolo diminuiscono, il terzo aumenta; e viceversa. Ciò significa che deve esistere una relazione fra i tre angoli del triangolo.

Si riesce a intuire quale sarà questa relazione considerando i casi “limite”, cioè sia il caso in cui il punto C va a cadere sulla base, sia il caso in cui lo stesso punto si allontana indefinitivamente da questa. Notiamo infatti che a mano a mano che il vertice C si avvicina alla base, gli angoli alla base tendono a zero mentre quello al vertice tende a un angolo piatto; la somma degli angoli tende perciò a un angolo piatto. Se invece il vertice C si allontana dalla base – e, col pensiero, distaccandosi ormai dall'esperienza materiale, possiamo immaginare che la sua distanza dalla base aumenti all'infinito – gli angoli alla base tendono ad angoli retti mentre quello al vertice tende a zero. Anche in tal caso, dunque, la somma degli angoli tende a un angolo piatto [1]. (Castelnuovo, 1964, p.89)



As it can be noticed, the use of the model (made from simple materials as a rubber band, cord and nails) is conceived as a way to discover the sum of the angles of a triangle through the study of many different triangles obtained by a continuous modification of ABC. The observation of the changing amplitudes leads to conjecture that there is a relation between the three angles, but it is only by the study of the limit cases (eventually imagining them) that it is possible to realize that the sum is exactly a straight angle.

## Intuitive geometry in the digital devices' era

In her works, Emma Castelnuovo stresses the fact that the usage of models differs from the drawing of the geometrical shapes because this last representation is fixed, while models can be handled, doing and undoing them, studying a potential infinity of different shapes (Castelnuovo, 1963; 1964; 2008). The limit of drawing seemed to be overcome when geometrical figures appeared on the computer screen and Dynamic Geometry Software (DGS), which can be found in the mathematics classrooms all over the world. With DGS we mean those kind of softwares which afford to create images of lines, shapes and eventually solid figures but also to move them or some of their parts. This software generally respects the axioms of Euclidean Geometry and allows compass and ruler constructions preserving the construction properties even through movements of the geometrical entities and their constitutive parts. Another common feature consists in the possibility to show the trace of objects' movement.

The first DGS was Cabri-Geometre, launched in 1988, but many different analogous softwares appeared on the market in many different languages (Sträßer, 2002). The usage of these kind of software spread

widely and fast in schools but also in research: At least for Geometry in secondary schools (grade 5 to 10), it is definitely the type of software of which research offers most insights about (Hollebrands et al., 2007).

Everybody appears to be very enthusiastic about the role DGSs can play in the process of teaching/learning geometry: Figures become continuously dynamic allowing for that kind of exploration which was recognized as impossible on paper. Hence, a DGS appears as a perfect alternative to Castelnuovo's models. Nowadays some DGS are also downloadable from the internet for free, so the effort made by the teacher to obtain this tool is very much smaller than the one needed to construct a model. Starting from these premises, the era of concrete models seems to arrive at its end, leaving the scene to digital devices.

Naturally, a question arises: from the didactical, pedagogical and cognitive point of view, are the use of material models and the use of DGS equivalent in the classroom activity?

This is a very big question, which would require the analysis of many piece of research and maybe many theoretical insights. Here we just want to share some thoughts, to make clear our position and in order to be more concrete as possible (avoiding simple speculation) we present below an example of class activity on which we develop further our discussion in the last section of this paper.

### An example: relationships between sides and angle of triangles

With the aim to make some reflection about the question posed in the last section, we describe and analyze two lessons in which a DGS (GeoGebra) is involved. The lessons are thought in a sixth grade and lasts for 50 minutes each one; the teacher is the second author of this paper. The aim of such classes is to study the relationships among the length of a triangle's sides and the amplitude of the opposite angles.

In the first lesson, pupils work in couples on the computer. Each couple has to work on GeoGebra following these tasks:

- 1- draw three points named A, B and C;
- 2- link the three points in order to get a triangle;
- 3- use the command  to point out the angles of the triangle with their name and measure (pay attention:  $\alpha$  has to be the angle with vertex in A,  $\beta$  has to be the angle with vertex in B,  $\gamma$  has to be the angle with vertex in C);
- 4- move A, B, C in order to make AB the longest side of the triangle and use the symbols =, <, > to order the amplitudes of  $\alpha$ ,  $\beta$ ,  $\gamma$ ;
- 5- move A, B, C in order to make BC the longest side of the triangle and use the symbols =, <, > to order the amplitudes of  $\alpha$ ,  $\beta$ ,  $\gamma$ ;
- 6- move A, B, C in order to make ABC an isosceles triangle ( $AB = BC$ ) and use the symbols =, <, > to order the amplitudes of  $\alpha$ ,  $\beta$ ,  $\gamma$ ;
- 7- move A, B, C in order to get ABC an equilateral triangle ( $AB = BC = CA$ ) and use the symbols =, <, > to order the amplitudes of  $\alpha$ ,  $\beta$ ,  $\gamma$ .

Two teachers are supervising the students' work, helping them when required although it would not be the first experience in the use of GeoGebra for these pupils. In particular, the activity of one couple of students is recorded: both the screen and students movements and speech are video-recorded as shown in figure (see Fig. 1).

The two students (M. and F.) use to be generally quite brilliant and talkative during mathematics classes but their attitudes show some differences: M. is very thoughtful while F. is used to be more instinctive and to show a more pragmatic approach during the learning process.

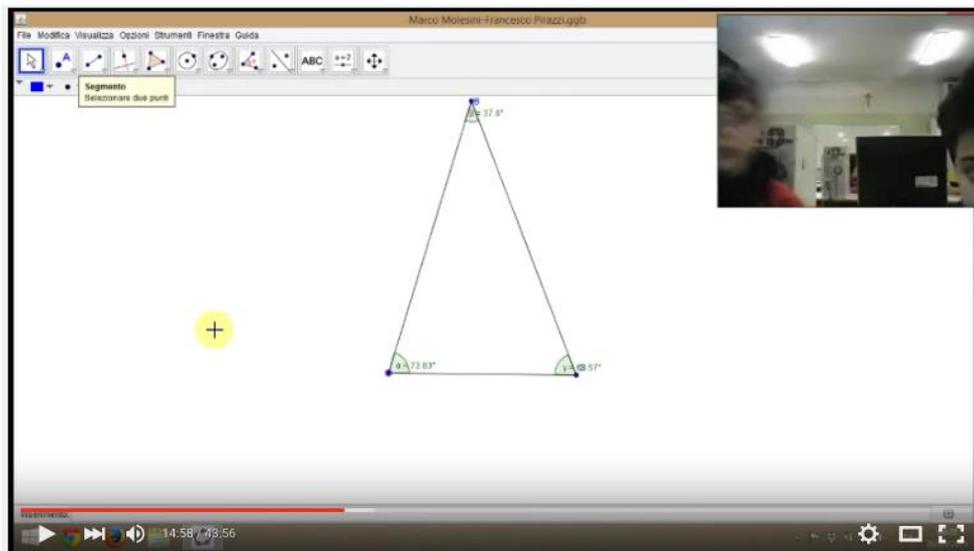


Fig. 1- M. and F. screen recording

During the first part of the activity F. holds the mouse of the computer; both students show some difficulties in finding the right commands in the software and F. tries to check randomly the functions of GeoGebra. In particular it is not a problem for them to draw the triangle ABC but they do not know

how to use the command . The first difficulty is related to the correct choice of the three points needed to draw the angle (the second point has to correspond to the vertex of the angle); the second difficulty is in the right choice of the order (clockwise or counterclockwise) to select the three points in order to avoid that the drawn angles is complementary to the desired one. When these obstacles are overcome by a trial and error process, M. and F. write on the paper the relation between the angles when the side AB is the longest one (task n° 4).

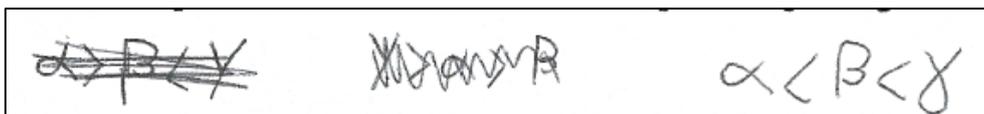


Fig. 2- M. and F. inscriptions on the paper

As shown in figure (see Fig.2), the use of symbols =, <, > changes gradually but students do not seem yet conscious that the only invariable relationship among angles (when AB is the longest side) is that  $\gamma$  is bigger than  $\alpha$  and  $\beta$  while there is not a fixed relationship between these last two angles.

A second interesting issue emerge from the work of M. and F.: a difficulty in drawing an isosceles triangle (task n° 6) using the software.

Pupils control the correctness of the drawn shape comparing the two angles that have to have the same amplitude. By the movement of the free points A, B, C in the plane they reach a *quasi-isosceles* triangle but the two angles continue to differ (in the decimal places of the amplitude) after each effort to fix the shape. When the boys seem to surrender to the impossibility to draw a perfect isosceles triangle, something happens:

M: It's not possible, anyway...

F: Let's call the teacher! .... Teacher! Please, come!

M: (talking to himself) ...or you can use a specific construction for isosceles triangles....

From that moment, the behaviours of the two students diverge drastically. F. is waiting for the teacher who is helping another couple of students while M. tries to follow a different strategy consisting in checking other commands of GeoGebra that could help to draw a “real” isosceles triangle. M.’s

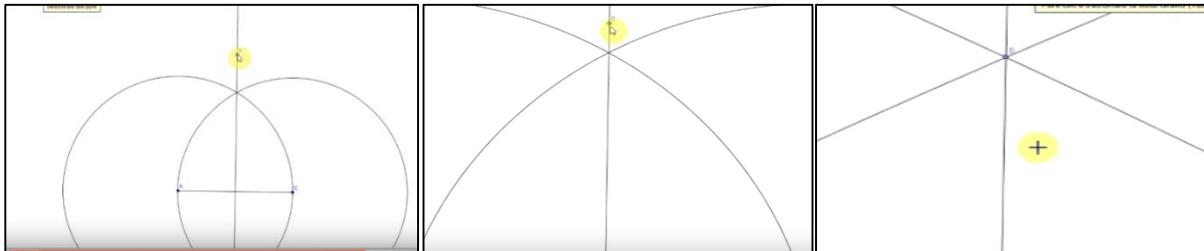
exploration of the software begin from the command  (compass): he points the cursor on one of the vertexes at the base of the triangle. This procedure recalls some similar strategy to draw specific triangles by the use of ruler and compass [2] on paper but M. is not able to go on with this strategy. Quite soon, another attempt brings to a more successful solution: M. remembers the use of the perpendicular bisector of a segment to control the symmetry of a shape and explores the commands of GeoGebra in a targeted

search of another strategy. When he finds the command (Perpendicular bisector ) he draws the perpendicular bisector of the base of the triangle and tries to put the third vertex on it. Even in this case the triangle is not “really” isosceles. This strategy is completed when the teacher suggests to M. and F. to draw again the shape starting from the perpendicular bisector of the base and *then* drawing the third vertex of the triangle as fixed on that perpendicular bisector.

In the case of the equilateral triangle (task n° 7) M. and F. choose a hybrid strategy: the third vertex of the triangle is fixed on the perpendicular bisector of a segment (that may show the awareness of the pupils that the equilateral triangle is a particular isosceles triangle); then they recover the use of the

command  to locate exactly the third vertex.

It is interesting to notice that the students do not know the existence of a specific way to fix a point on the intersection of two curves, so they resort to the expedient of progressively zooming the image to be sure the point falls in the “exact” position, as shows in figure (see Fig. 3).



**Fig. 3 - M. and F. use of the zoom**

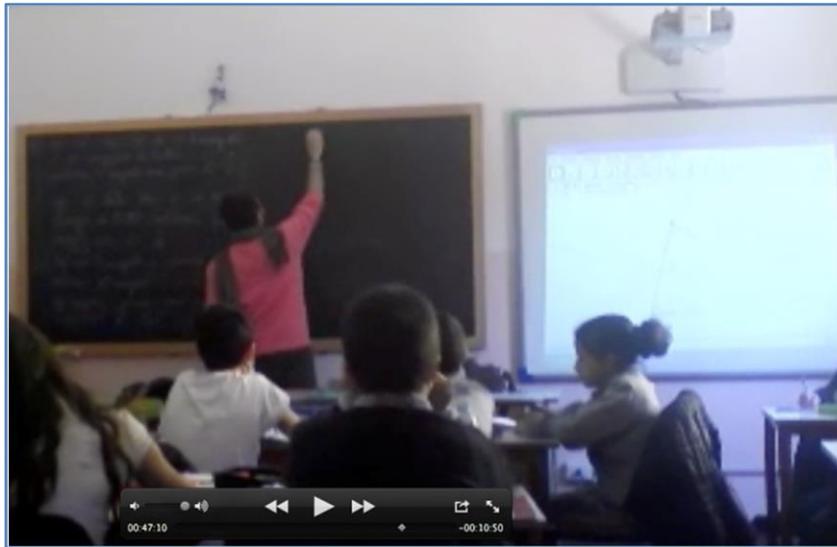
The second lesson takes place in the classroom one week after the previous one. A traditional blackboard and an interactive one equip the room. The interactive screen is used to recall the different steps of the work done on GeoGebra by the couples in the computer lab; the blackboard is used by the teacher to fix shared ideas and conclusions emerging from students during the discussion, as shown in figure (Fig. 4). The whole lesson (lasting one hour) is video-taped from the backside of the classroom.

The recalling of the phases of the previous activity allows to share conclusions and to compare representations; in particular relationships as  $\alpha < \beta < \gamma$  and  $\gamma > \beta > \alpha$  are identified as equivalent while different solutions, as  $\alpha < \beta < \gamma$  and  $\beta < \alpha < \gamma$ , are found by different groups of students. These differences let emerge the possibility of different kinds of shapes under the same condition (AB is the longest side of the triangle). The first shared conclusions written on the blackboard are then:

“If AB is the longest side, than the angle  $\gamma$  is the biggest one” and “If BC is the longest side, than the angle  $\alpha$  is the biggest one”.

The comparison of the statements brings to a more general conclusion:

“The longest side opposes the widest angle”.



**Fig. 4 - The teacher transcribes the shared results on the blackboard**

Even if this lesson has the aim to lead the class to this kind of shared conclusion, the simultaneous use of the traditional blackboard and the software allows for further possibilities of explorations. In particular the need to consider a limit case, during the discussion on the isosceles triangle, emerge by a girl (G).

G: (referring to the vertex C, Fig. 5) Excuse me, teacher, could you bring the vertex closer to the opposite side?

T: (moving the vertex C towards AB) In this way?

G: Yes.

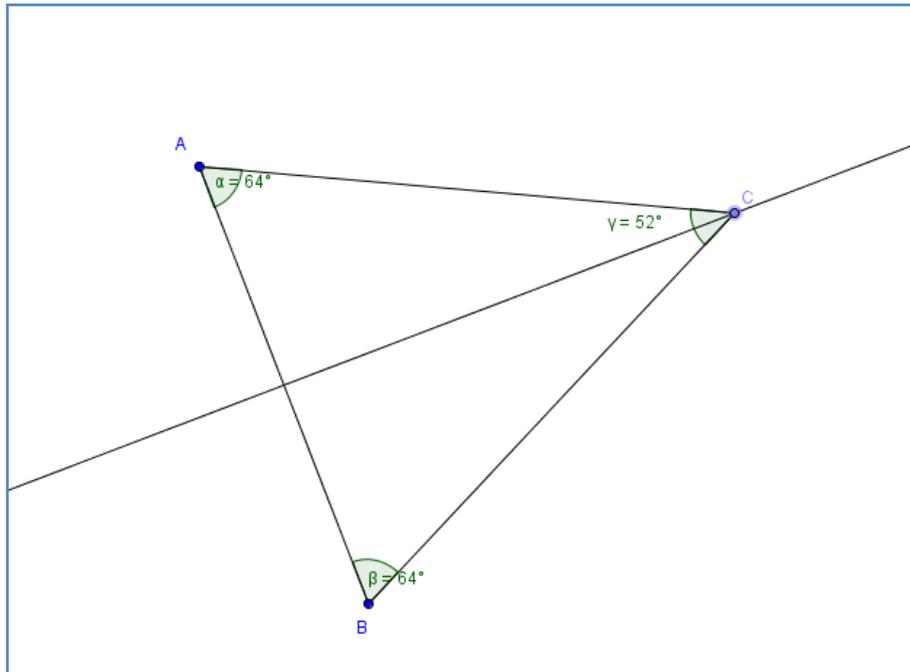
T: (changing the position of the vertex C till the triangle collapses on a single line) Is it still an isosceles triangle?

Everybody: Yes

T: Well, in all these cases the triangle is still an isosceles triangle because we constructed it as isosceles.

The last part of the lesson is spent to analyse the case of the equilateral triangle; finally the conclusion of the whole activity is written on the blackboard in order to complete the previous statement:

“The longest side opposes the widest angle; equal sides oppose equal angles”.



**Fig. 5 – The drawing shown on the interactive whiteboard during the class discussion. The straight line is the perpendicular bisector of the side AB. C is fixed upon the straight line while A and B are free points.**

## Discussion and Conclusion

As promised above, in this section we share some thoughts about possible differences and similarities between a classroom activity in which a DGS is involved and Intuitive Geometry by Castelnuovo. A brief analysis of the lessons presented in the last section helps in organizing the discourse.

Looking at the activity of M. and F. using GeoGebra to solve the proposed tasks, we can recognize the explorative dimension that was promoted in the book “Intuitive Geometry”. The two boys work on a specific drawn triangle that can be modified continuously; in this way it represents an entire family of triangles (those with a side longer than the others, the isosceles ones, the equilaterals ones). The fact that the inscriptions made by pupils change over time shows how their exploration is fructuous in terms of mathematical thinking. During the second lesson, the possibility to share and discuss the results obtained by the couples allows to notice what is common in different works. So, the initial sentences (which refers mainly to the particular construction analysed by the couple) became more and more abstract until all the class reached together a general final statement.

M. and F. use many commands of the software to explore the task. In particular they use the “zoom” command to see what “really” happens when the lines intersect each other. This kind of exploration is possible in every DGS but would be quite difficult (if not impossible) with physical model. The “zoom” command seems to have the potential to convince the students about the difference between a “real” isosceles triangle and a “quasi-isosceles”. This appears to be a good feature of DGSs that is not shared by concrete models. On the other side, Emma Castelnuovo (1964) says that is in the limitation given by concreteness that the child feels the need for the mathematical abstractedness, but if the software does not give limits (apparently you can zoom until you want), will the pupil still feel the need of a more abstract mathematical model?

According to Andrà and Santi (2011), “Intuition can be seen as the sensuous side of intellectual-emotional activity when the activity is mediated mainly through objects, artefacts, gestures, bodily movements, deictic and generative use of natural language”. These researchers (as many others)

recognize the importance of the involvement of the body (including movement and gestures) in the intuitive activity. Does the interaction with the mouse of the computer give the same perceptual-motor experience, which is given by a concrete model? The answer seem to be negative. Let's think about the concrete model proposed in the quotation of the first section: It is quite similar to the construction used by M. and F. to explore the case of the isosceles triangle. But, in the case of the concrete model there is a rubber band opposing to a pull. The physical properties of the rubber can give insights about the mathematical properties of the obtained shapes (the triangle with the maximum perimeter requires the maximum strength in pulling the rubber band). This kind of feedback is not given by the DGS. On the computer screen everything is mediated by the sense of sight, the other ones are never involved and this appears as a limit for the intuition. For example, the resistance of the elastic band can suggest the limit case in which the vertex of the isosceles triangle collapses on the base. It sounds reasonable to think that this case is not naturally considered in using a DGS. The analyzed data says that this is not necessarily the case: G. looks at the screen of the interactive whiteboard and she asks the teacher to explore that particular limit case.

In conclusion, DGSs can be used within a *continuous* and *active* method as advocated by Castelnovo (Furinghetti, Menghini, 2014). This kind of software allows exploration that would be not possible with a concrete model (the “zoom” command). But, while watching the screen, the students use just their sight; the other senses are not involved. In particular, physical feedbacks (as the pull of the rubber band) can be perceived just through the usage of concrete objects. This does not mean that the students would not feel the need to explore the limit cases, which were pointed as fundamental by Castelnovo. While drawing on DGSs, the students transfer their knowledge about constructions through compasses and rulers; so this activity appears as a very important one in terms of educational goals. On the other side, it is easy to imagine how strong can be the role of the knowledge about geometrical properties while constructing a concrete model of a shape.

Finally, we can say that some features of Intuitive Geometry can be implemented in the activity with DGS. However, the activity with concrete models differs from the software mediated one. It does not mean that one of the two is better than the other: They can be both used to achieve important educational goals but the teacher has to be aware of the differences. In fact, one kind of proposal does not exclude the other; the teacher can choose one or the other according to the particular educational aim of the moment.

There is no claim that concrete models and dynamic instruments may be replaced by their digital copies without loss. Trivially, the digitalization of instruments allows them to become widely available: where there is an access to the Internet one can play with these models interactively. Yet a deep analysis of the changes (if any) in both didactical and cognitive processes when a concrete object is replaced by a digital copy is yet to be performed. (Bartolini et al, 2010, p.30)

The Italian Commission for Mathematical Instruction stresses that “the meaning cannot reside only in the tool neither it can emerge only through the interaction between the student and the tool. The meaning resides in the aims which the tool is used for, in the plans which are elaborated to use the tool” (UMI-CIIM, 2001). For any kind of technology (older and newer ones) it is true what Arzarello and colleagues (2002) say: It is a misbelief to think “if the technology used is good, then didactics will certainly improve”.

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## Notes

1. Put two nails A and B on a little table with chosen sizes (fig. 177). A rubber band passes around the nails and one part of the band is pulled through a cord, which is fixed in the middle point, perpendicularly to the line connecting the nails (which is realized through the other part of the rubber band). In this way, many isosceles triangles with fixed base AB are obtained and the vertex C can vary. These triangles share the same base while the height can vary when the strength used to pull the rubber band changes. It is interesting to study the angles of these triangles while the height varies. If we imagine to begin from the biggest triangle which can be made on the tablet and then to leave more and more the cord, then the angle in C will become bigger and bigger while the angles at the base A and B become smaller and smaller: So if two angles of a triangle decrease, the third increases; and viceversa. That means that there is a relation between the three angles of the triangle.

We can perceive by intuition this relation by considering the “limit” cases: the case in which the point C falls on the base and the case in which that point goes far from the base. Indeed, we notice that as more the vertex C gets closer to the base, the base's angles go to zero while the one in the vertex tends to a straight angle; then the sum of the angles tends to a straight angle. If the vertex C goes far from the base – and with our thought, detaching from the material experience, we can imagine that its distance from the base goes to infinity – the base's angles tend to right angles while the one in the vertex tends to zero. Also in this case the sum of the angles tends to a straight angle.

2. In the Italian school, it is common for pupils to learn the drawing of basic geometric entities and shapes with the use of ruler and compass during Technology classes in 6<sup>th</sup> grade.

## References

- Andrà, C., & Santi, G. (2011). A semiotic characterization of intuitions. In B. Ubuz (Ed.) *Proceedings of the 35th Conference of the International Group for Psychology of Mathematics Education*. Vol. 4, pp. 113-120. Ankara, TR.
- Arzarello, F., Olivero, F., Paola, D., & Robutti, O. (2002). A cognitive analysis of dragging practices in Cabri environments. *ZDM*, 34(3), 66-72.
- Bartolini Bussi, M.G., Taimina, D., & Isoda, M. (2010). Concrete models and dynamic instruments as early technology tools in classrooms at the dawn of ICMI: from Felix Klein to present applications in mathematics classrooms in different parts of the world. *ZDM*, 42(1), 19-31.
- Castelnuovo, E. (1963). *La didattica della matematica*. Firenze: La Nuova Italia. Translations: Castelnuovo, E. (1968). *Didaktik der Mathematik*. Frankfurt am Mein: Akadem. Vlg. Gesell. Castelnuovo, E. (1970). *Didáctica de la Matemática moderna*. Mexico: Trillas.
- Castelnuovo, E. (1949-1964). *Geometria intuitiva*. Firenze: La Nuova Italia
- Castelnuovo, E. (2008). In F. Lorenzoni (Ed.), *L'officina matematica: ragionare con i materiali*. Molfetta: La Meridiana.
- Furinghetti, F., & Menghini, M. (2014). The role of concrete materials in Emma Castelnuovo's view of mathematics teaching. *Educational Studies in Mathematics*, 87, pp.1-6.
- Hollebrands, K., Laborde, C., & Sträßer, R. (2007). The learning of geometry with technology at the secondary level. In M. K. Heid & G. Blume (Eds.), *Handbook of Research on Technology in the Learning and Teaching of Mathematics: Syntheses and Perspectives*. Greenwich, CT: Information Age Publishing.
- Sträßer, R. (2002). Research on dynamic geometry software (DGS) — an introduction. *ZDM*, 34(3), 65-65.
- UMI-CIIM (2001). *Matematica per il cittadino – Matematica 2001*.



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Received November 16, 2015; revised November 29, 2015; accepted December 08, 2015

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